BEMT Algorithm for the Prediction of the Performance of Arbitrary Propellers

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This report describes details of the Blade Element Momentum Theory (BEMT) and its application for the calculation of the performance of any arbitrarily shaped propeller.

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Executive Summary

Blade Element Momentum Theory is a useful tool for the prediction of any arbitrary propeller's performance, provided that the geometry of the propeller blade and other relevant information on the operating conditions of the propeller are given. The method is described in detail, including a description of the algorithm that may be used as a guide for writing a computer program for the calculation of a propeller's thrust and torque for any combination of propeller's operating altitude, forward speed, rotational speed and blade pitch setting. The geometry of the propeller should be given in terms of the coordinates of points on the surface of aerofoils or blade elements, which make up the whole propeller. The slope of the lift coefficient and the drag coefficient of each blade element should be calculated using any proven CFD computer software or be measured in a wind tunnel, and given as inputs to the BEMT computer program.

Acknowledgements

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Symbols and Abbreviations

Symbol/ Anagram	Description
α	Angle of attack of airflow seen by the blade element
α_0	Zero lift angle of attack of the aerofoil section of a blade element
β	Blade pitch angle setting
γ	Tangent of this angle is same as the ratio of drag to lift coefficient of a local blade element
θ	Local pitch angle of a blade element
σ	Blade solidity
ϕ	Local inflow angle
Ω	Propeller rotational speed in radian per second
a	Axial inflow factor
b	Swirl factor
c	Chord of a local blade element
$\dot{m}$	Mass flow rate of air passing through the propeller disc
r	Radial distance from the hub axis to the location of a blade element
A	Area of the propeller disc
B	Number of blades making up the propeller
C_d	Drag coefficient of a local blade element
C_l	Lift coefficient of a local blade element
$C_{l,\alpha}$	Slope of the lift curve of a blade element
R_{tip}	Radius of the propeller
R_{hub}	Radius of the hub to which the propeller is attached
T	Propeller's thrust
Q	Propeller's torque
$\frac{dT}{dr}$	Propeller's thrust grading
$\frac{dQ}{dr}$	Propeller's torque grading
V	Forward speed of propeller
V_0	Axial component of the airflow velocity seen by a blade element
V_T	Same as V_0
V_s	Axial component of airflow velocity within the streamtube enveloping the propeller disc, far to the rear of the propeller.
V_Q	Azimuthal component of the airflow velocity seen by a blade element
V_R	Incoming airflow seen by a blade element

1 INTRODUCTION

An example of a real engineering problem is the desire to have the capability to predict the aerodynamic performance of a propeller, if the propeller's geometry as well as its operating conditions, are specified.

The propeller is assumed to be somehow or other fixed in space, immersed in the flow, thus the flow over the propeller is not interfered with in any way by the presence of another body even though the propeller in actual fact is attached to an aircraft. The aircraft may be stationary with the engine running at full throttle, driving the propeller at its maximum rotational speed, while the aircraft is preparing to commence its run along the runway prior to taking off. On the other hand the aircraft may be cruising at a certain altitude at a certain forward speed, while the propeller is rotating at a certain rotational speed.

The more general situation of the case where the aircraft is accelerating, climbing up, turning around or moving sideway as well as forward will not be considered due to the complexity of the airflow that would exist in such situations. Even with these simplififying assumptions applied, the problem is still formidably difficult.

The governing equations for the motion of viscous, compressible fluids are the well known Navier-Stokes equations, which are very highly non-linear second order partial differential equations and impossible to solve analytically. Even if the flow is steady, e.g. the airflow where an aircraft is moving forward at a constant speed in a perfectly stationary or stagnant atmosphere, there are regions very close to the surface of the aircraft where the boundary layer that is formed may be turbulent and moving chaotically as a function of time (e.g. see ref. 2, 3, 4 and 5). If the flow remains attached to the body, and the boundary layer doesn't separate anywhere in the flow field, then the airflow outside of the thin boundary layer may be assumed to behave as an inviscid flow. By applying the assumption of inviscid flow, the highly complex Navier-Stokes equation can be simplified to become the Euler equation. The flow is then divided up into 2 distinct regions, namely the viscous boundary layer region and the inviscid outer region. Within the thin boundary layer region the flow is governed by the viscous Navier-Stokes equation, which can be simplified into the Boundary Layer equation by taking advantage of the fact that the length scale in the direction normal to the flow is very much less than length scales in the other direction(s), and making the allowable simplifications based on that fact.

The flow over a 2-dimensional wing can then be solved by solving the Euler equation as the governing equation, and then applying a boundary layer correction to the results obtained, to take care of the fact that within a very thin region adjacent to the wing's surface the flow is actually viscous and must be treated as such.

Even though the Euler equation is much simpler than the Navier-Stokes equation, its solution is by no means easy. Regions of surface discontinuities or shock waves can be formed in a compressible flow, provided the speed of the flow in some regions is greater than the speed of sound within the flow field. The possible presence of shock wave(s) within the flow field makes it very difficult to solve the Euler equation even by utilizing numerical techniques.

A better way of simplifying the Navier-Stokes is to remove its time dependence by applying certain time averaging technique, such as the statistical averaging technique proposed by Reynolds. The simplified Navier-Stokes equation, which is obtained from this procedure, is known as the Reynolds Averaged Navier-Stokes or RANS equation. This equation is very similar to the Euler equation, except for the fact that it contains a Reynolds turbulent stress term as a new variable. To enable closure of the problem of the RANS equation, where there are more variables than the number of equations available to be solved, it is necessary to develop turbulence modelling so that empirical equation(s) can be added on to the basic governing equations. The RANS solver is capable of giving a very good prediction of the required quantities of the viscous, compressible flow, depending on the quality of the turbulence model utilized. The numerical methods used in solving the RANS equation are basically dimilar to the ones utilized in solving the Euler equation.

If the flow is always subsonic everywhere within the flow field, the complexity of the governing equation can be simplified considerably. For such a situation the governing equation for the model flow is the incompressible RANS or incompressible Euler equations plus incompressible boundary layer equation. The governing equation can be further simplified by assuming that the flow is irrotational. The compressible Euler equation is simplified into the Full Potential Equation by applying the irrotationality condition. For the incompressible flow case, the simplified governing equation is now the Laplace equation, which is a linear second order partial differential equation.

Another problem that must be faced when dealing with real flows is the fact that almost all flows are 3-dimensional. Without going into the details, it is sufficient to state here that a 3-dimensional flow is considerably more difficult to solve than the corresponding 2-dimensional flow. The basic reason for this is the fact that the generation of the computational grid for 3-dimensional flow is far more complicated than that for its 2-dimensional counterpart.

The flow over a 3-dimensional wing is considerably more difficult to solve numerically than that for the flow over 2-dimensional aerofoil. To simplify the analysis of a 3-dimensional flow over a wing, it is assumed that the wing can be thought of as being composed of a large number of wing sections, which are obtained when a wing is cut up by a large number of planes normal to the wing span axis. Each wing section or element can be thought of as being a very small part of an infinitely long 2-dimensional aerofoil of the same cross-sectional shape. The flow over each wing section is assumed to be independent of what is happening elsewhere over any of the other section. In other words, it is assumed that the flow is locally 2-dimensional. However, in reality the flow is strongly 3-dimensional with a cross-flow happening along the wingspan. Therefore, in reality the flow over a wing section is highly dependent or influenced by whatever is happening elsewhere along the wing. This 3-dimensionality effect must then be reintroduced into the flow field using a suitable modelling technique. A method that has been found to be quite effective in modelling a 3-dimensional flow is to assume that a vortex sheet is formed by the wing and is shed at the trailing edge of the wing.

The vortex sheet induces a downwash to the incoming flow, hence reducing the effective angle of attack of the flow. This in turns reduces the lift and creates a lift dependent drag, which is known as the induced drag.

The Lifting Line Method is based on the above assumptions and can be applied with reasonable results to investigate the aerodynamic properties of 3-dimensional wing. This method is fast and can give reasonably accurate prediction for wings that have no sweep back angle, not too thick, without dihedral or anhedral, and is not highly tapered.

A similar concept can also be applied to investigate the aerodynamic properties of a propeller. A propeller blade is similar to an aircraft wing, except for the fact that it rotates rather than moving forward as in the case of the wing. The vortex sheet produced by the blade is shed at its trailing edge. However, due to the fact that the blade is rotating then the shed vortex sheet would have a shape that resembles a helix, rather than just a flat sheet as in the case of the flying wing. The complex shape of the vortex sheet makes it very difficult to analyze the flow field induced by the vortex sheet, by utilizing the fundamental equation for a vortex known as the Biot-Savart equation.

Fortunately the gross aerodynamic properties of the idealized propeller can be analyzed using the momentum theory of an actuator disc. The momentum equation can be combined with the aerofoil or blade element theory, in a BEMT (blade element momentum theory) method, which is extremely useful in predicting the aerodynamic performance of a "rotating wing", such as a propeller, helicopter rotor or a wind turbine rotor.

In the following sections we will discuss in great detail the assumptions made in developing the BEMT method, and the application of the theory in predicting the thrust, torque and aerodynamic efficiency of a propeller. The discussion is sufficiently detailed to include the development of an algorithm for the prediction of the aerodynamic properties of a propeller, which can be easily translated into a working computer program to numerically compute those properties if the propeller's geometry and its operating conditions are given as data.

2 BLADE ELEMENT THEORY

A propeller consists of a small number of identical blades attached axisymmetrically to a common hub. Each propeller blade may be thought of as being similar to a high aspect ratio wing, thus the analysis of a propeller may borrow some familiar ideas from the aerodynamic analysis of an aircraft wing.

The geometry of a wing is described in terms of its planform or its overall shape when viewed from above. The wing's overall length from tip to tip is called its span, while its width is called chord, which usually varies along the span. The cross section, obtained when a plane cuts a wing normal to its span axis, is known as an aerofoil. In general the aerofoil shape and chord length vary along the span. The chord is the straight line connecting the nose (most forward point) to the tail (most rearward point) of the aerofoil.

The pattern of airflow around a wing is rather complicated, but basically may be described as follows. Let the flow be from left to the right, normal to the wing span axis or the quarter chord line. As an air particle accelerates around the curved aerofoil shape, its speed increases. The greater curvature of the aerofoil upper surface, compared to the lower one, results in the speed of the air particles above the aerofoil being faster than underneath it. Bernoulli equation states that as the speed increases the static pressure decreases, thus the air pressure acting on the aerofoil upper surface is less than the pressure on its lower surface. The pressure difference acting on the aerofoil is the reason why the wing is capable of producing lift and must suffer drag.

Due to the presence of the pressure difference between the lower and the upper aerofoil surface, the air particle will tend to flow from the lower surface to the upper surface. However, it is obvious that air cannot penetrate the wing surface and hence such a flow is not possible except at the wing tip, where air is free to move around the tip from the lower to the upper surface of the wing. This air motion has an overall effect of creating a cross flow from the wing root to the wing tip along the wing lower surface, and from the wing tip to the root along the wing upper surface. Due to this cross flow, the air is not merely accelerated but also imparted with a rotational motion. The initially irrotational free stream flow becomes rotational as the flow goes around the wing. Therefore the wing may be modeled as a bound vortex along its span and in addition a vortex sheet is shed downstream from the trailing edge of the wing. The trailing vortex sheet is basically unstable, and eventually rolls up forming a very powerful vortex line at the tip of the wing, known as the trailing tip vortex.

The tip vortex induces a downward velocity component to the incoming airflow, thus decreasing its effective angle of attack when passing over the aerofoil. The angle of attack is defined as the angle subtended by the direction of airflow relative to the aerofoil chord line. In turn the reduction of the effective angle of attack decreases the lift acting on the wing. At the same time it also produces a component of drag, which is dependent on the lift produced, known as the induced drag. This is in contrast to a 2-D wing where the span is infinitely long and the size and shape of its cross section are the same all along the span. The 2-D wing, or infinitely long aerofoil, does not have trailing vortices and hence no induced drag. All the drag associated with a 2-D wing is caused by viscosity or skin friction and pressure distribution around the aerofoil.

The aerodynamic analysis of a 2-D wing is far simpler than a corresponding one for that of a 3-D wing. The evaluation of the lift and drag coefficients for a 2-D wing are thus also far simpler than those for a 3-D wing. Moreover, from the above discussion it is obvious that the lift of a 3-D wing is less than that for a 2-D wing, whilst the opposite is true for drag. A wing with a rectangular planform (constant spanwise chord distribution) and the same cross section shape all along its span, but with a finite span length, is referred to as a finite length aerofoil. The argument above shows that the finite aerofoil has less lift and greater total drag per unit span length compared to an infinite aerofoil. The reason for this is that for the finite aerofoil there is a cross flow along the span, which in turns is responsible for the formation and shedding of the trailing vortex sheet at the wing's trailing edge. The vortex sheet, which moves downstream, is basically unstable and rolls up as a strong trailing tip vortices at both tips of the wing. The strong tip vortex then induces a downwash velocity, which deflects the incoming airflow at the wing section, and thus reduces the effective angle of attack as described previously. It follows, therefore, that the aerodynamic properties of an aerofoil is dependent on its span length. To be more precise it depends on the aspect ratio, a non-dimensional quantity defined as the aerofoil span length divided by its average chord length.

Generally speaking a finite wing may have a non-rectangular planform and may be twisted both geometrically as well as aerodynamically. A wing may be twisted geometrically such that the geometric angle of attack distribution of the airflow over the wing varies along the span. Furthermore, even if the wing is not twisted geometrically the distribution of aerodynamic properties along the span may vary because the shape of the aerofoil varies along the span. This is known as aerodynamic twist. The aerodynamic properties of a finite wing is thus determined by a number of factors, such as the wing geometry (planform, aspect ratio, aerofoil shape, and twist distribution) as well as the angle of attack, Reynolds Number and Mach Number of the flow. The complicated aerodynamic analysis of a 3-D wing can be simplified by assuming that the flow over any small element of the span (an aerofoil of elemental width) is basically 2 dimensional locally, thus we can apply the results of 2-D aerodynamic analysis. The 3-D effect is taken into account by considering the spanwise cross flow, which is modeled as a vortex sheet shed by the wing's trailing edge. This in turn induces a downwash velocity, which varies along the span and is dependent on the geometry of the 3-D wing. By assuming that the lift distribution, hence vortex strength distribution, along the span is given by a Fourier series, then the vortex strength at any point along the span can be calculated provided the coefficients of the series are known. An equation involving the series coefficients can be generated, which is applicable at any particular point along the span. If the equation is applied at N points then a set of N simultaneous equations can be obtained, which can be solved to give the values of the N Fourier series coefficients. This method is known as the lifting line theory. More detailed discussion on the Lifting Line Theory has been given by a number of authors, such as Schlichting, Truckenbrodt and Ramm (ref.2), J.D. Anderson, Jr. (ref.3), Katz and Plotkin (ref. 4) and Houghton and Carpenter (ref.5)

For a propeller flow the problem is a little bit more complicated, because the "wing" or propeller blade is not simply moving forward but rotating about its longitudinal axis.

It is assumed that the propeller is attached to an aircraft that is moving forward along its longitudinal axis. Therefore, the lifting line theory, which is applicable for the analysis of an aircraft wing, must be modified accordingly and is known as the blade element theory. The application of this theory for the analysis of the performance of helicopter rotor has been discussed by Stepniewski and Keys (ref.6) and Leishman (ref.7) among others.

On the plane of the propeller, each small element of the blade can be regarded as an aerofoil with an infinitesimal width of Δr , where r is the radial distance from the hub axis or the centre of propeller rotation. For a blade element, located at a radial distance between r and $r+\Delta r$, the tangential or azimuthal speed of the air seen by the blade is given by Ωr, where Ω is its angular speed in radian per second. If the propeller blade is spinning at a speed of n revolution per minute (rpm), then the angular speed is given as follows

$$\Omega = 2\pi n / 60 \quad \text{rad/sec}$$

The propeller is moving axially with a speed of *V*, which is the forward speed of the aircraft, at the same time as it is spinning around. Since the axial direction is normal to the plane of the propeller disk, therefore the resultant velocity seen by the blade element, V_R, is given by the following expression

$$V_R = \sqrt{V^2 + (\Omega r)^2}$$

In the above discussion it is assumed that the blade element has no effect whatsoever on the airflow. Obviously this is not true. From the previous discussion on the flow around an aircraft wing we know that the aerofoil would produce lift, ΔL, and drag, ΔD, which can be combined into a resultant force as follows

$$\Delta F_R = \sqrt{\Delta L^2 + \Delta D^2}$$

This elemental resultant force can be split into 2 components, namely an axial component, ΔT , and an azimuthal component, $\Delta Q / r$. The azimuthal component when multiplied by radial distance r is the elemental torque acting on the blade, ΔQ , whilst the axial component is actually elemental thrust produced by the blade. This implies that there is an incremental axial velocity induced by the propeller blade, which can be assumed to be a small fraction of the forward speed of the aircraft or propeller, *aV*, where *a* is a small number less than 1. Similarly, there must also be a reduction in the azimuthal velocity component, which gives rise to the elemental torque. This azimuthal speed decrement must be related to and be a small fraction of the azimuthal velocity, -$b\Omega r$, and is in the direction opposing the azimuthal speed. Note that *b* is a small positive number. The basic problem is the fact that the radial distribution of *a* and *b* values are unknown and must be computed. This in essence is the central problem of the application of BEMT method.

From the above discussion it follows that the axial velocity seen by the blade element is actually (1+ *a*)*V*, whereas the azimuthal velocity is (1-*b*)Ωr. The angle subtended by those 2 velocity components is known as the inflow angle, ϕ , and is given in terms of the resultant velocity as follows

$$V_R = \sqrt{(1+a)^2 V^2 + (1-b)^2 (\Omega r)^2}$$

$$\tan\phi = \frac{(1+a)V}{(1-b)\Omega r}$$

$$\sin\phi = \frac{(1+a)V}{V_R} \quad \text{or} \quad V_R = V(1+a)\operatorname{cosec}\phi$$

$$\cos\phi = \frac{(1-b)\Omega r}{V_R} \quad \text{or} \quad V_R = \Omega r(1-b)\sec\phi$$

It can be seen from the above, that if the inflow angle is known then it is easy to compute the values of the factors a and b.

A sketch of a propeller blade, the total radius of which is *R*, is shown in figure 1.

A diagram showing the flow direction and the forces acting on a blade element is given in figure 2, while a sketch of the airflow passing over the propeller disk is shown in fig.3.

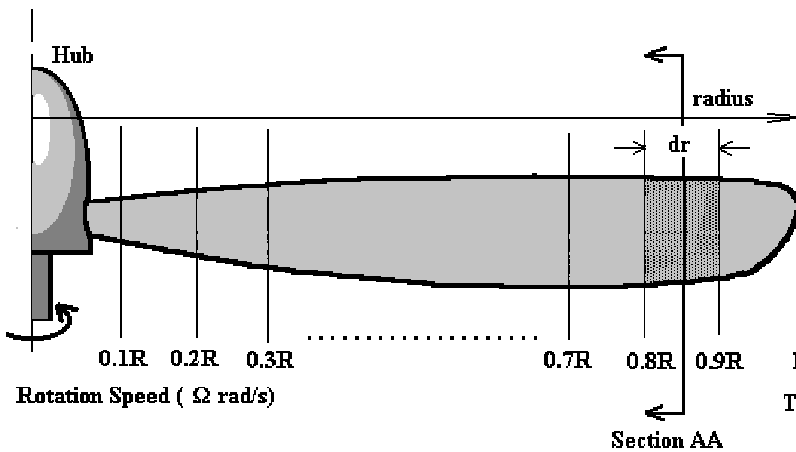

Fig.1 The blade of a propeller (from ref.1)

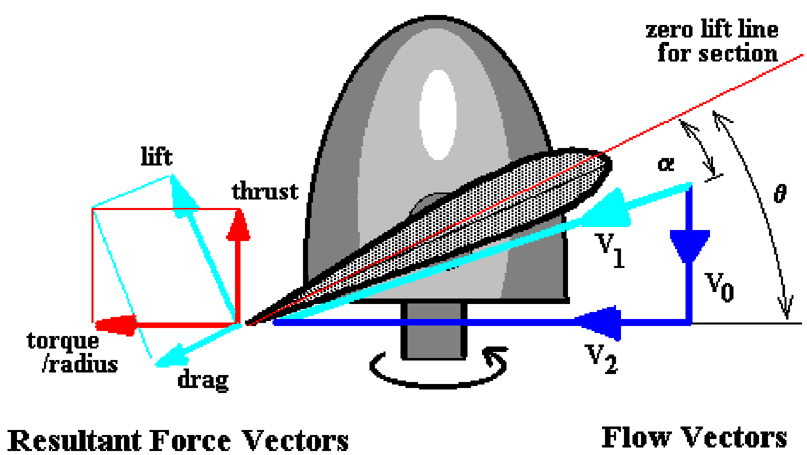

Fig.2 Definition of flow and force directions on a blade element (from ref.1)

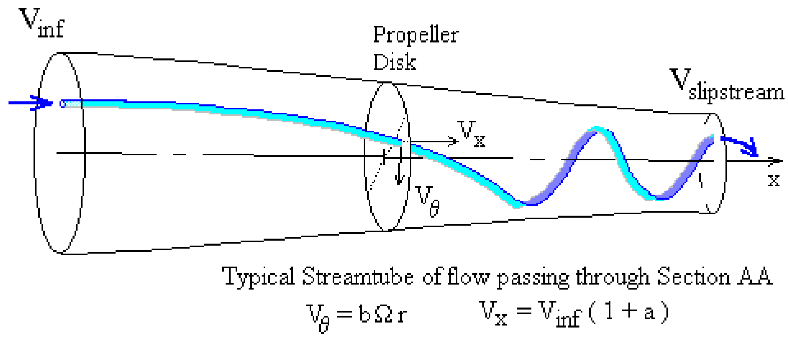

Fig.3 The streamtube of the flow that passes over the propeller disc (from ref.1)

The essence of the blade element theory is to reduce the difficulty of modelling a complex 3-dimensional flow by assuming that it can be replaced by a linear summation of a large number of simpler 2-dimensional flows. The blade is divided up into a number of small elements and the flow over each blade element is assumed to be independent of each other. Therefore, the aerofoil or 2-dimensional wing theory can be applied to analyse the flow over each blade element. The 3-dimensional effect is then modelled by assuming that there is a vortex sheet being shed at the trailing edge of the blade. As the vortex sheet moves further downstream, it follows a helical path. The downwash speed induced by the vortex sheet is too difficult to derive analytically, due to the complexity of its path. Nevertheless the 3-dimensional effect is modelled by assuming that as the flow approaches the propeller disc plane, it is affected by an induced velocity in both axial and azimuthal directions. The change in flow direction as the fluid approaches the propeller disc plane is described in terms of an axial and an azimuthal inflow factors. The momentum theory and the aerofoil theory are then applied to determine the magnitude of the inflow factors. In the following sections we will discuss the evaluation of the inflow factors in greater detail.

3 MOMENTUM THEORY

From the discussion in the previous section we know that a central problem in the application of the aerofoil theory for propeller performance analysis is how to determine the spanwise distribution of the inflow angle, ϕ. In this section the momentum theory is discussed and applied to help solve the problem of finding the inflow angle distribution.

The momentum theory was first proposed by Rankine (ref.8) in 1865, for the evaluation of the performance of idealized propellers. The details of the propeller geometry are not taken into account and completely neglected. The propeller is instead idealized as an actuator disk. The airflow pattern due to the operation of the propeller is modelled as a streamtube flow, as shown in fig.3. Upstream of the propeller disc the diameter of the streamtube is somewhat larger than the disc diameter, which then contracts to the diameter of the disc as the flow moves downstream passing through the disc, and continues to contract as it moves further downstream. At the upstream section, the free stream velocity is the speed V of the aircraft, to which the propeller is attached. The flow is accelerated as it moves downstream, reaching a speed of V_0 as it passes through the disc, and eventually settling at a speed of V_s at a sufficiently far downstream location.

While the air velocity is regarded as changing gradually from V to V_s, the static pressure of the flow is idealized as being discontinuous as the flow crosses the actuator disc. Just upstream of the disc the air pressure is p_1 and it suddenly jumps to p_2 immediately downstream of the disc.

The force acting on the actuator disc due to the pressure difference is equal to the thrust imparted by the propeller, T, and is equal to the pressure difference multiplied by the disc area, A.

$$T = A(p_2 - p_1)$$

$$A = \pi R^2$$

The radius R is equal to half of the propeller diameter, D.

The thrust imparted to the air by the propeller is also equal to the mass flow rate within the streamtube multiplied by the difference between air velocity leaving and that entering the tube, i.e.

$$T = \dot{m} \; (V_s - V)$$

$$\dot{m} = \rho V_0 \, A$$

Equating the expressions for thrust, T, we get the following result

$$p_2 - p_1 = \rho V_0 (V_s - V)$$

Since the flow from the forward free stream section to immediately upstream of the disc is of constant enthalpy, therefore Bernoulli equation applied to this region gives the following result

$$p_0 + \tfrac{1}{2}\rho V^2 = p_1 + \tfrac{1}{2}\rho V_0^2$$

Similarly, for the region downstream of the disc we have

$$p_2 + \tfrac{1}{2}\rho V_0^2 = p_0 + \tfrac{1}{2}\rho V_s^2$$

Combining the 2 Bernoulli equations above, we get the following

$$p_2 - p_1 = \tfrac{1}{2}\rho(V_s^2 - V^2)$$

Comparing the above with the expression for the pressure difference obtained earlier finally gives the following result

$$\rho V_0(V_s - V) = \tfrac{1}{2}\rho(V_s^2 - V^2)$$

The above equation can be simplified further with the following result

$$V_0 = \tfrac{1}{2}\left(V_s + V\right) \qquad \text{or} \qquad V_s = 2V_0 - V$$

If the propeller is attached to a stationary aircraft, e.g. an aircraft that is revving up prior to taking off, then $V = 0$, and thus

$$V_s = 2\ V_0$$

If the aircraft is cruising, then V is not zero and we can define an inflow factor, a, as follows

$$a = (\ V_0 - V\ )/\ V$$

$$V_0 = V\,(1 + a)$$

It is obvious that, the definition of a, is only valid if V is not zero. When V is not zero, the quantity $aV = V_0 - V = \tfrac{1}{2}(V_s - V)$ is the axial velocity induced by the propeller disc. On the other hand if the propeller is stationary, the velocity induced by the propeller is simply $V_0 = \tfrac{1}{2}V_s$ and a is quite meaningless.

Rankine momentum theory was modified slightly by W. Froude (ref.9) in 1878 to take into account the fact that the value of the inflow factor is in general not constant across the whole face of the actuator disc. The inflow factor is a function of radial distance from the axis of the propeller, and is actually determined by the geometry of the propeller.

The actuator disc may be considered as being made up of a very large number of rings or annuli, the width of which is constant and is equal to Δr, and in the limit it can be made to be infinitesimal with a value of dr. This situation is depicted in figure 4.

The momentum theory is now applied for an "elemental streamtube" or annulus whose cross section area is $2\pi\, r\, dr$ at the radial station *r*, rather than on the whole area of the actuator disc. The result of such an analysis is similar to the previously obtained result, namely that for a stationary propeller the axial flow velocity induced by the propeller is $V_0 = \frac{1}{2} V_s$, whereas for a forward moving propeller it is given by $aV = V_0 - V = \frac{1}{2}(V_s - V)$.

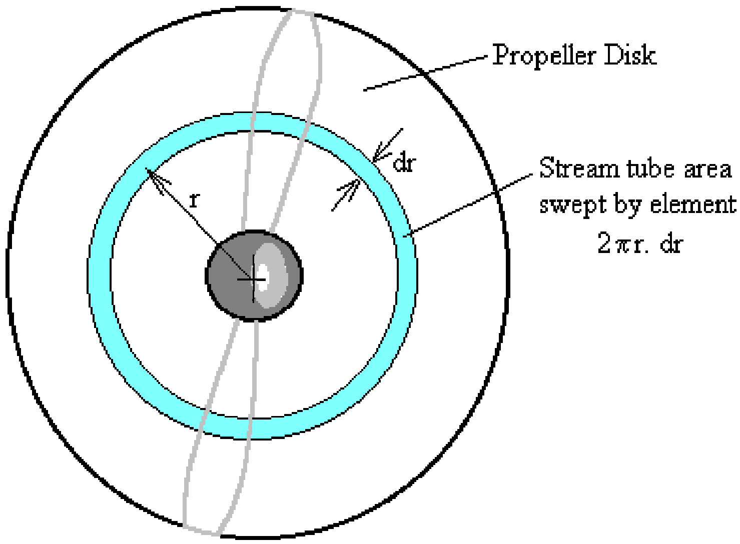

Fig.4 The propeller as an actuator disc (from ref.1)

The elemental thrust, which is equal to the axial momentum increment through the annulus, is given by the following

$$\triangle T = \rho V_0\, 2\pi r \triangle r \left(V_s - V\right)$$

If $\triangle r$ is taken to be infinitesimal, the above expression can be simplified to give the equation for thrust grading as follows

$$\frac{dT}{dr} = 4\pi\, \rho\, r V^2\, a\left(1 + a\right)$$

If the propeller is stationary, the equation must be changed as follows

$$\frac{dT}{dr} = 4\pi\, \rho\, r V_0^2$$

The theory can be refined further by noting that a vortex sheet would be shed at the trailing edge of the blade, similar to the concept of lifting line theory for the finite lifting wing described in the previous section. However, for the propeller case the "wing" is rotating, hence the influence of the vortex sheet on the main flow is far more complicated. Without the presence of the vortex sheet, a small elemental streamline would simply move axially. However, the vortex sheet would induce an additional speed upstream of the disc and a reduction of speed downstream of the disc in the azimuthal direction or the direction of rotation of the propeller blade. The induced azimuthal speed must be a small fraction of the rotational velocity, given by $b\Omega r$, where b is a small positive value and Ω is the blade rotational speed in radian per second as described in the previous section. The streamline of the flow is no longer simply in the axial direction. After passing through the propeller disc the streamline would have both azimuthal as well as axial velocity components, and the path of the moving fluid particle would describe a helical shape as shown in figure 3. Furthermore, the airflow is subjected to an azimuthal speed jump of $2b\Omega r$. More detailed discussion about this azimuthal speed jump is given by Houghton and Carpenter (ref.5). A simplified explanation for this azimuthal speed jump can be given as follows.

Consider the flow of a fluid around a stationary aerofoil. If the flow were truly inviscid, the fluid particles would regain its undisturbed free stream velocity not far immediately to the rear of the aerofoil's trailing edge. However, if the flow is viscous then the friction due to viscosity would slow down the speed of the fluid particles, which move on the surface of the aerofoil. Since real fluid flows are always viscous, therefore there will always be a speed reduction after the fluid flows over the aerofoil. This also applies for the flow over a propeller blade element and is given by the factor b. This factor is then related to the momentum loss in the azimuthal direction. This in turn is related to the drag force experienced by the blade and thus is related to the torque required to rotate the blade. Furthermore, this azimuthal speed reduction also has an effect on the magnitude of the inflow angle, ϕ. Whilst the value of b may actually be rather small, the results of propeller analysis using the theory would be less accurate if the factor b were altogether neglected. According to Houghton and Carpenter (ref.5) the momentum loss in the azimuthal direction is given by the mass flow rate multiplied by $2b\,\Omega\, r$, and when multiplied by r we can obtain the expression for the elemental torque, dQ, as follows

$$dQ = \rho V_0 . 2\pi r\, dr . 2b\Omega r . r$$

Therefore, the equations for torque grading for the forward moving and stationary propellers are as follows

$$\frac{dQ}{dr} = 4\pi\rho r^3 \Omega V \left(1+a\right) b$$

$$\frac{dQ}{dr} = 4\pi\rho r^3 \Omega V_0\, b$$

It is obvious that the momentum theory alone is insufficient to determine the values of the inflow factor, a, and the swirl factor, b. In the following section we will discuss how the momentum theory can be combined with the blade element theory, which can then be applied to evaluate the values of a and b.

4 BLADE ELEMENT MOMENTUM THEORY

In this section the blade element theory and the momentum theory will be combined in a blade element momentum theory, so that the values of the inflow factors a and b can be computed.

It is assumed that the geometry of the aerofoil sections all along the span of the blade is completely specified. This means that the aerodynamic properties of every blade element can be calculated using 2-D aerodynamic CFD software, such as Fluent, or be measured in a wind tunnel. Therefore, the values of lift and drag coefficients of all aerofoil sections are known as functions of the angle of attack, α , which generally speaking varies as a function of radial distance r, due to the geometric and aerodynamic twist of the blade as well as the fact that the chord also varies radially.

Let us now consider a particular blade element at radial station r. The chord of the aerofoil is at an angle of θ to the plane of the propeller disc. This angle is known as the local pitch angle and its value varies as a function of radial distance. As the blade rotates with an angular speed of Ω, the blade element is subjected to elemental lift and drag forces of $\triangle L$ and $\triangle D$ respectively. The values of those forces are given by

$$\triangle L = \tfrac{1}{2}\rho V_R^2 \, C_l \, c \triangle r$$

$$\triangle D = \tfrac{1}{2}\rho V_R^2 \, C_d \, c \triangle r$$

where ρ is the fluid density, c is the aerofoil chord, C_l and C_d are the lift and drag coefficients respectively and V_R is the resultant velocity of the fluid as seen by the aerofoil.

The resultant velocity seen by the blade element has 2 components, namely the axial velocity $V_T = V_0 = V(1+a)$, and an azimuthal component, $V_Q = (1-b)\Omega r$. Therefore, the resultant velocity is

$$V_R^2 = V_T^2 + V_Q^2$$

The inflow angle, ϕ, is now defined as follows

$$\tan\phi = \frac{V_T}{V_Q} = \frac{V_0}{\Omega r(1-b)} = \frac{V(1+a)}{\Omega r(1-b)}$$

The effective angle of attack of the flow relative to the aerofoil is

$$\alpha = \theta - \alpha_0 - \phi$$

where α is the effective angle of attack, θ is the local pitch angle, α_0 is the zero lift angle of attack of the aerofoil shape and ϕ is the inflow angle. The blade is normally mounted on a mechanism, which is attached to the hub, and can be rotated about an axis perpendicular to the hub. Therefore, if the blade is rotated through an angle of β then all the local pitch angles will be increased (or decreased) by the angle β . The angle β is known as the blade pitch angle, and its value is normally given relative to a reference value at a particular radial station, which is usually designated to be the station at 70 percent of the blade radius. It follows, therefore, that the effective angle of attack should be defined to include the blade pitch angle as follows

$$\alpha = \beta + \theta - \alpha_0 - \phi$$

All values on the right hand side of the above equation are known except for the inflow angle, ϕ. If it is known then the effective angle of attack can be calculated. Since the geometry and angle of attack of each aerofoil is known, then any suitable CFD method may be used to compute the lift and drag coefficients. The lift coefficient is a linear function of the angle of attack as follows

$$C_l = C_{l,\alpha} . \alpha$$

where $C_{l,\alpha}$ is the slope of the lift versus angle of attack curve, i.e. or $C_{l,\alpha} = \frac{\partial C_l}{\partial \alpha}$, whose value is only dependent on the shape of the aerofoil. Furthermore, the above equation is valid only for small angle of attacks, when the flow is not separated.

The elemental drag force is along the line of the flow as seen by the aerofoil (see fig.2), whereas the elemental lift force is normal to that direction.

An angle γ can be defined as follows

$$\tan \gamma = \frac{C_d}{C_l}$$

The elemental resultant force can now be written as follows

$$dF_R = \sqrt{dL^2 + dD^2} = dL\sqrt{1 + \tan^2 \gamma} = dL \sec \gamma$$

$$dF_R = \tfrac{1}{2} \rho V_R^2 \, C_l \, c \, dr \; \sec \gamma$$

Note that c is the chord of the aerofoil.

The resultant force can be resolved into 2 components, with one component being in the axial direction and the other in the azimuthal direction. The axial component of the force is the same as the elemental thrust force due to a single blade, and if the propeller has B blades then the elemental thrust of the propeller is

$$dT = dL.\cos \phi - dD.\sin \phi = \tfrac{1}{2} \rho V_R^2 \, B \, c \, dr \left(C_l \cos \phi - C_d \sin \phi \right)$$

$$dT = \tfrac{1}{2} \rho V_R^2 \, B \, c \, C_l dr \left(\cos \phi - \tan \gamma \sin \phi \right)$$

Now it is noted that

$$\cos \phi - \tan \gamma . \sin \phi = \sec \gamma \left(\cos \gamma \cos \phi - \sin \gamma \sin \phi \right) = \sec \gamma \cos \left(\phi + \gamma \right)$$

Therefore the thrust grading can be written as follows

$$\frac{dT}{dr}=\tfrac{1}{2}Bc\,\rho V_R^2\,C_l\,\sec\gamma\cos(\phi+\gamma)$$

The azimuthal component of the elemental resultant force when multiplied by the radial distance, *r*, is the elemental torque of the propeller. Using a similar argument as for the thrust grading, it can be shown that

$$\frac{dQ}{dr}=\tfrac{1}{2}Bc\,\rho V_R^2\,C_l\,r\sec\gamma\sin(\phi+\gamma)$$

Equating the thrust grading and torque grading above with the ones obtained from the momentum theory, we get the following equations.

$$\tfrac{1}{2}Bc\,\rho V_R^2\,C_l\,\sec\gamma\cos(\phi+\gamma) = 4\pi\,\rho\,r V_T\left(V_T - V\right)$$

$$\tfrac{1}{2}Bc\,\rho V_R^2\,C_l\,\sec\gamma\sin(\phi+\gamma) = 4\pi\,\rho\,r V_T b\Omega r$$

For the stationary propeller case, $V = 0$, and since $V_T = V_R.\sin\phi$, therefore the thrust equation can be simplified as follows

$$\sigma C_l\,\sec\gamma\cos(\phi+\gamma)=4\sin^2\phi$$

The blade solidity factor σ is defined as follows

$$\sigma=\frac{B.c}{2\pi r}$$

A function $f(\phi)$ is now defined as follows

$$f(\phi)=4\sin^2\phi-\sigma C_l\sec\gamma\cos(\phi+\gamma)$$

The value of ϕ can be obtained by solving the transcendental equation $f(\phi)=0$. This equation can not be solved analytically and must be solved numerically. There are quite a few numerical methods that can be used to solve the problem (e.g. see ref.12 and ref.13). Some of these methods are very efficient, e.g. Newton Method and Secant Method, and some are not very efficient, e.g. the Method of Successive Substitution. All of these methods are based on a repetitive procedure, which is known as the iterative technique. A solution is guessed, and the guessed solution is then used to calculate a new guessed value, which is hoped to be closer to the solution when compared to the previous guessed value. One complete set of calculations, which ends up with a new guessed solution, is called an iteration. After a number of iterations it is desirable that the computed value should have converged to the required solution. However, it is possible that the iteration never converge and may in fact diverge away from the sought for solution. It is necessary, therefore, to ensure that the method chosen should always converge. Some of the methods for solving transcendental equations will be discussed in the next section. It is sufficient to say here that the equation can be solved to give the required value of ϕ.

The torque equation can be developed in a similar manner as follows

$$\tfrac{1}{2} Bc\,\rho V_R^2\, C_l\ \sec\gamma \sin\left(\phi+\gamma\right) = \ 4\pi\,\rho\, r\, V_R \sin\phi\, b\Omega r$$

The above equation can be simplified as follows

$$\frac{b\Omega r}{V_R} = \tfrac{1}{4}\sigma\, C_l\ \sec\gamma \operatorname{cosec}\phi \sin\left(\phi+\gamma\right)$$

However, from the definition of ϕ it is also known that

$$\frac{\left(1-b\right)\Omega r}{V_R} = \cos\phi$$

Combining the 2 equations above, we get

$$\frac{b}{\left(1-b\right)} = \tfrac{1}{4}\sigma\, C_l\ \sec\gamma\ \sec\phi\ \operatorname{cosec}\phi\ \sin\left(\phi+\gamma\right) = K_1$$

Since the values of all the terms on the right hand side of the above equation are known, therefore the value of K_1 can be computed.

The value of *b* can now be computed as follows

$$b = \frac{K_1}{1+K_1} = \frac{1}{1+\dfrac{4\cos\gamma\sin\phi\cos\phi}{\sigma C_l\sin\left(\phi+\gamma\right)}}$$

The value of V_R and V_T can then be calculated as follows

$$V_R = \left(1-b\right)\Omega r \sec\phi$$

$$V_T = V_0 = V_R \sin\phi$$

For the more general case where the propeller is moving forward at a speed of *V*, the equations for thrust and torque grading are a bit more complicated and are developed below.

The equation for thrust is

$$\tfrac{1}{4}\sigma V_R^2 C_l\ \sec\gamma\cos\left(\phi+\gamma\right) = a\left(1+a\right)V^2 = aV\,V_R\sin\phi$$

The inflow factor, *a*, is defined as follows

$$a = \frac{V_0}{V} - 1 \quad \text{or} \quad V_0 = \left(1+a\right)V = V_R\sin\phi$$

The thrust equation can then be simplified as follows

$$aV = \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \cos(\phi+\gamma).V_R$$

Similarly, the equation for torque gives the following result

$$b\Omega r = \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \sin(\phi+\gamma).V_R$$

Since $V_0 = V_T = (1+a)V = V_R \sin\phi$, therefore

$$V_R \sin\phi = V + \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \cos(\phi+\gamma).V_R$$

which can be simplified as follows

$$F(\phi) = \frac{V}{V_R} = \sin\phi - \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \cos(\phi+\gamma)$$

Similarly, from the torque equation we can derive the following result

$$V_R \cos\phi = \Omega r - \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \sin(\phi+\gamma).V_R$$

which can be simplified as follows

$$G(\phi) = \frac{\Omega r}{V_R} = \cos\phi + \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \sin(\phi+\gamma)$$

Since $V_R = \dfrac{V}{F(\phi)} = \dfrac{\Omega r}{G(\phi)}$, therefore $V.G(\phi) = \Omega r.F(\phi)$ .A function $g(\phi)$ can now be defined as $g(\phi) = \left(F(\phi).\Omega r - G(\phi).V\right).\sin\phi = 0$ or

$$g(\phi) = (\Omega r \sin\phi - V\cos\phi)\sin\phi - \tfrac{1}{4}\sigma C_l \sec\gamma\left(\Omega r\cos(\phi+\gamma) + V\sin(\phi+\gamma)\right)$$

The equation $g(\phi) = 0$ is a transcendental equation in ϕ that can be solved for the required value of ϕ. This equation must be solved numerically as mentioned previously for the stationary propeller case.

Once the value of ϕ has been found, then the values of the other quantities can be computed easily as follows

$$V_R = \frac{V}{F} = \frac{\Omega r}{G}$$

$$a = \frac{\sin\phi}{F} - 1$$

$$b = 1 - \frac{\cos\phi}{G}$$

Using the above method, we can predict the values of the thrust and torque gradings at any radial station, provided the geometry of the propellers blade is known.

The expressions for thrust and torque gradings of a stationary propeller are as follows

$$\frac{dT}{dr} = 4\pi \rho r \left(V_R \sin\phi\right)^2$$

$$\frac{dQ}{dr} = 4\pi\rho r^3 b\Omega V_R \sin\phi$$

For a forward moving propeller the expressions are as follows

$$\frac{dT}{dr} = 4\pi \rho r V^2 a\left(1+a\right)$$

$$\frac{dQ}{dr} = 4\pi\rho r^3 \Omega V \left(1+a\right) b$$

In order to calculate the thrust produced by the propeller and the torque required to turn the propeller, the blade must be divided up into a large number of blade elements, each of equal width. The blade section does not start from the axis, but from the hub, which is quite often located at a radial distance of 0.2R, if the radius of the blade is R.

Let us assume that the blade is divided up into N elements, with each element's width being given by the following formula

$$\vartriangle R = \frac{R - 0.2R}{N}$$

The aerofoil shape of the mid-section of the blade element is assumed given. For any particular value of the blade pitch, β, the value of the thrust and torque gradings can be computed using the method described above. The thrust and torque of the propeller are given by the following integrals

$$T = \int_{0.2R}^{R} \frac{dT}{dr}.dr$$

$$Q = \int_{0.2R}^{R} \frac{dQ}{dr}.dr$$

Since the values of the thrust and torque gradings are known only at a finite number of locations, each integral must be replaced by a numerical integration such as the trapezoidal rule or other suitable methods. A very simple but reasonably accurate approximation is given as follows, provided N is reasonably large

$$T = \vartriangle R.\sum_{n=1}^{N}\left(\frac{dT}{dr}\right)_n$$

$$Q = \Delta R.\sum_{n=1}^{N}\left(\frac{dQ}{dr}\right)_n$$

If the blade is divided up into N equal width elements, and the aerofoil shape at the edge, rather than at the mid section, of each element is given, then the following trapezoidal rule should be used instead of the previous approximation

$$T = \Delta R\left[\tfrac{1}{2}\left(\left(\frac{dT}{dr}\right)_1 + \left(\frac{dT}{dr}\right)_{N+1}\right) + \sum_{n=2}^{N}\left(\frac{dT}{dr}\right)_n\right]$$

$$Q = \Delta R\left[\tfrac{1}{2}\left(\left(\frac{dQ}{dr}\right)_1 + \left(\frac{dQ}{dr}\right)_{N+1}\right) + \sum_{n=2}^{N}\left(\frac{dQ}{dr}\right)_n\right]$$

It should be noted that station 1 is at the blade root, whereas station (N+1) is at the tip of the blade, where both the thrust and torque gradings may be assumed to be zero. Therefore, the above equations can be simplified as follows

$$T = \Delta R\left[\tfrac{1}{2}\left(\frac{dT}{dr}\right)_1 + \sum_{n=2}^{N}\left(\frac{dT}{dr}\right)_n\right]$$

$$Q = \Delta R\left[\tfrac{1}{2}\left(\frac{dQ}{dr}\right)_1 + \sum_{n=2}^{N}\left(\frac{dQ}{dr}\right)_n\right]$$

The input power required to produce the torque is given by

$$P_{in} = \Omega Q = 2\pi\frac{n}{60}Q$$

The power output is

$$P_{out} = T.V$$

Therefore the theoretical efficiency of the propeller is

$$\eta_T = \frac{T.V}{\Omega Q}$$

5 NUMERICAL SOLUTION OF A TRANSCENDENTAL EQUATION

In this section we will discuss some methods for solving a complicated transcendental equation. This type of equation is not possible to solve analytically and the only recourse to obtaining a solution is by means of numerical methods, which generally speaking always involves a repetitive procedure known as iteration.

An example of a complicated transcendental equation is the equation derived in the previous section for the case of the stationary propeller.

$$f(\phi) = 4\sin^2\phi - \sigma C_l \sec\gamma \cos(\phi + \gamma)$$

$$\sigma = \frac{Bc}{2\pi r} \text{ (a constant independent of } \phi\text{)}$$

$$C_l = C_{l,\alpha}.\alpha \text{ (} C_{l,\alpha} \text{ is a constant independent of } \phi\text{)}$$

$$\alpha = \theta^* - \phi$$

$$\theta^* = \beta + \theta - \alpha_0 \text{ (a constant independent of } \phi\text{)}$$

$$\tan\gamma = \frac{C_d(\alpha)}{C_l(\alpha)}$$

The equation is applicable for a blade element located at a radial distance *r*. The aerofoil shape at that location is given so that the lift coefficient slope $C_{l,\alpha}$ and the drag coefficient as a function of angle of attack α, or $C_d(\alpha)$, are known. Therefore, the angle γ can be calculated.

The blade solidity σ is a known constant since *B* is the number blades and *c* is the known chord of the aerofoil at the given radial distance *r*.

The local pitch angle θ, and the zero lift angle of the aerofoil α_0 are known function of radial distance *r*, thus the angle θ^* can be calculated easily for any given blade pitch setting angle β.

The basic problem is to find the solution for the equation $f(\phi) = 0$.

The problem is impossibly difficult to solve unless we have some idea of the range of values in which the solution may lie. Furthermore, it would help a great deal if it is known that within a certain range of values of ϕ there is only one solution that satisfies the equation. For the problem discussed here, the physical situation that is represented by the equation can be utilized to simplify the problem. For example, it is known that the inflow angle is always greater than zero but less than 90 degrees. This is quite a large range, but it does restrict the location of the required solution. Furthermore, the physics of the problem does tell us that there is only one solution that lies within the given range.

For the given problem it is assumed that the solution is within the range of

$$0 < \phi < \pi / 2$$

In the simplest method of solution, the value of the solution is guessed, say at the mid point of the given range or $\phi = \pi / 4$. The equation is now rewritten as follows

$$\sin\phi = \tfrac{1}{2}\sqrt{\sigma C_l \sec\gamma \cos(\phi + \gamma)}$$

A new value of ϕ is then computed using the above equation, substituting the guessed value of ϕ into the right hand side of the equation, and using the previously given equations to calculate the values of C_l and γ. From the computed value of $\sin\phi$, a new guessed value of ϕ can be calculated. If this new value of ϕ is the same or very close to the previously guessed value of ϕ, then it is assumed that the latest computed value of ϕ is the sought for solution. This method is very simple but there is no guarantee that the solution will always converge. In fact the guessed value may diverge further and further away from the true solution.

There are other methods, which converge faster than the Method of Successive Substitution, e.g. the Newton Method or the Method of Secant. Newton method requires the computation of the derivative of $f(\phi)$, which for this case is unknown. The Secant Method does not require the value of the derivative, but not as efficient as the Newton method. Both methods belong to the group of methods that are known as the Open Methods (see ref. 14) and do not always converge to the required solution.

There are other methods that always converge and known as the Bracketed methods. Two of the better known bracketed methods are the Regula Falsi and the Bisection Methods. More detailed discussions on all of the methods are given in ref. 12, 13 and 14. Only the regula falsi and bisection methods will be discussed here.

The Regula Falsi method or Method of False Position can be described as follows.

First it is noted that for $\phi = 0$ the value of $f(0) = f(\phi = 0)$ can be calculated with the following result

$$f(0) = 4\sin^2 0 - \sigma C_l \sec\gamma \cos(\gamma) = -\sigma C_l \text{ (negative value)}$$

Similarly, for $\phi = \pi / 2$ we get the following result

$$f(\pi/2) = 4\sin^2(\pi/2) - \sigma C_l \sec\gamma \cos(\pi/2 + \gamma) = 4 + \sigma C_l \tan\gamma$$

Noting that $\tan\gamma = \dfrac{C_d}{C_l}$ therefore

$$f(\pi/2) = 4 + \sigma C_d \text{ (positive value)}$$

Let us now define the following

$$f_{neg} = f\left(\phi_{neg}\right) \text{ (-ve)} \quad \text{and} \qquad f_{pos} = f\left(\phi_{pos}\right) \text{ (+ve)}$$

From the results of the previous computation, it can be concluded that the initial guess should be as follows

$$\phi_{neg} = 0 \text{ so that } f_{neg} = f\left(\phi_{neg}\right) = -\sigma C_l$$

$$\phi_{pos} = \pi / 2 \text{ so that } f_{pos} = f\left(\phi_{pos}\right) = 4 + \sigma C_d$$

It should be noted that σ, C_l and C_d are all positive.

It can be seen that f_{neg} has a negative value, while f_{pos} has a positive value. It follows, therefore, that somewhere between $\phi = 0$ and $\phi = \pi / 2$ there must be a value where the curve of $f\left(\phi\right)$ crosses the horizontal axis and thus have a value of 0.

A line drawn passing through the points (ϕ_{neg} , f_{neg}) and (ϕ_{pos} , f_{pos}) must cut the axis nearer to the solution than either ϕ_{neg} or ϕ_{pos}. The value of ϕ_{new}, where the line cuts the axis, is then given by the following equation

$$\phi_{new} = \phi_{neg} - \frac{\triangle \phi}{\triangle f} . f_{neg}$$

$$\triangle \phi = \phi_{pos} - \phi_{neg}$$

$$\triangle f = f_{pos} - f_{neg}$$

The value of $f_{new} = f\left(\phi_{new}\right)$ is then calculated using the expression for the known function $f\left(\phi\right)$. The bracketed range of values where the required solution may lie is now narrowed as follows

If f_{new} is negative, the old values of ϕ_{neg} and f_{neg} are replaced by ϕ_{new} and f_{new}. On the other hand, if f_{new} is positive, then it is the old values of ϕ_{pos} and f_{pos} that should be replaced by ϕ_{new} and f_{new} respectively.

The procedure is then repeated by computing a newer value of ϕ_{new}, until such a time when the value of $\left|f_{new}\right| = \left|f\left(\phi_{new}\right)\right|$ is less than a predetermined small positive value, ε. The latest computed value of ϕ_{new} is then the required solution. Another criterion for stopping the iteration is when the latest computed value of ϕ is sufficiently close to the previously computed value of ϕ, such that another iteration is not going to change the guessed value of ϕ by very much.

Since in this method the range of values for the solution is bounded on both the negative and the positive sides, it follows that the method will always converge. It should be noted that in this method the solution range may not be reduced by very much, since it is possible that only one particular side of the limiting boundary is shifted closer and closer to the solution.

A method where the solution range is forced to become closer and closer, so that we can specify the number of iterations to get a desired interval where the solution may lie, is called the Bisection or Interval Halving Method.

In the Bisection Method, we start with known negative and positive limiting boundaries of the bracketed range where the solution is, similar to the regula falsi method.

Therefore, we start with the following

$$\phi_{neg} = 0 \text{ so that } f_{neg} = f\left(\phi_{neg}\right) = -\sigma C_l$$

$$\phi_{pos} = \pi/2 \text{ so that } f_{pos} = f\left(\phi_{pos}\right) = 4 + \sigma C_d$$

A new value or guessed value is taken to be the mid point of the interval

$$\phi_{new} = \tfrac{1}{2}\left(\phi_{neg} + \phi_{pos}\right)$$

The value of $f_{new} = f\left(\phi_{new}\right)$ is then computed using the given equations as described previously in the description for the regula falsi method. The interval is now bisected or halved in size by shifting either the negative or the positive limiting boundary of the interval depending on whether f_{new} is positive or negative.

If f_{new} is negative, then replace the old values of the negative limiting boundary $\phi_{neg} = \phi_{new}$ and $f_{neg} = f_{new}$. On the other hand if f_{new} is positive, then replace the old values of the positive limiting boundary, $\phi_{pos} = \phi_{new}$ and $f_{pos} = f_{new}$.

The procedure is then repeated N number of times, at the end of which the width of the interval has been halved N times or reduced by a factor of $\left(\tfrac{1}{2}\right)^N$. For example, after 10 iterations the width of the interval where the solution lies in this example is $\left(\pi/1024\right) = 0.00307$. After 20 iterations the interval is narrowed down to 0.000003. Thus in this method we can specify the width of the final interval that we want the solution to be bracketed in. The solution can in fact be much more accurate than the final size of the interval if it is calculated using the method of regula falsi, i.e. it is computed to be the point of intersection of the line joining the points $\left(\phi_{neg}, f_{neg}\right)$ and $\left(\phi_{pos}, f_{pos}\right)$ of the latest interval, with the horizontal axis.

Another method that may be faster than either the regula falsi or the bisection method is the Modified Regular Falsi Method. This is basically the regular falsi method. In the pure regular falsi method, one of the bounds may become stagnant, i.e. repeated computation of f_{new} always gives negative value or conversely it always returns a positive value. In this case only one side of the interval brackets is always shifted, while the other side is stagnant or stays put in the same place. This may not be desirable because it may cause the convergence to slow down considerably. To overcome this problem, the method should be modified as follows. Ideally the interval boundary points should be shifted alternately following each iteration. If it turns out that a boundary is moved twice in 2 successive iterations, while the other boundary remains stagnant, then in the next iteration the new guessed solution is computed using the point $\left(\phi_{stag}, \frac{1}{2} f_{stag}\right)$ rather than $\left(\phi_{stag}, f_{stag}\right)$, where $\left(\phi_{stag}, f_{stag}\right)$ is the stagnant limiting boundary. This technique can improve the solution rate of convergence quite significantly, at least for some type of problems (e.g. see ref.14).

There are many other possibilities for the modified regula falsi method but they will not be discussed here. It is sufficient to mention in passing here that the bisection method and the regula falsi method can be applied alternatively, and in some type of problems this compound method will actually converge faster than either the pure bisection or pure regula falsi method.

The methods described above are not only applicable for the stationary propeller case, but also for the case where the propeller is moving forward. For this case the equation to be solved is as follows.

At each radial position, *r*, the values of $\sigma, \theta^*, C_{l,\alpha}, \alpha, C_d, C_l$ and γ can all be evaluated as for the previous case of stationary propeller.

The inflow and swirl factors, *a* and *b*, are given by the following equations

$$a = \frac{\sin\phi}{F(\phi)} - 1 \quad \text{and} \quad b = 1 - \frac{\cos\phi}{G(\phi)}$$

$$F(\phi) = \frac{V}{V_R} = \sin\phi - \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \cos(\phi + \gamma)$$

$$G(\phi) = \frac{\Omega r}{V_R} = \cos\phi + \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \sin(\phi + \gamma)$$

The equation that must be satisfied is $V_R = \dfrac{V}{F(\phi)} = \dfrac{\Omega r}{G}$, which can also be rewritten as $g(\phi) = 0$, where $g(\phi)$ is defined as follows

$$g(\phi) = \left(F(\phi) - \frac{V}{\Omega r} . G(\phi) \right) \sin\phi$$

It should be noted that $F(\phi)\sin\phi$ and $G(\phi)\sin\phi$ can be simplified as follows

$$F(\phi) = \sin\phi - \tfrac{1}{4}\sigma C_l \sec\gamma \operatorname{cosec}\phi \left(\cos\phi\cos\gamma - \sin\phi\sin\gamma\right)$$

$$F(\phi) = \sin\phi - \tfrac{1}{4}\sigma C_l \left(\frac{\cos\phi}{\sin\phi} - \tan\gamma \right) = \sin\phi - \tfrac{1}{4}\sigma \left(C_l \frac{\cos\phi}{\sin\phi} - C_d \right)$$

Therefore

$$F(\phi)\sin\phi = \sin\phi\left(\sin\phi + \tfrac{1}{4}\sigma C_d\right) - \tfrac{1}{4}\sigma C_l \cos\phi$$

Similarly it can be shown that

$$G(\phi)\sin\phi = \cos\phi\left(\sin\phi + \tfrac{1}{4}\sigma C_d\right) + \tfrac{1}{4}\sigma C_l \sin\phi$$

and thus

$$g(\phi) = \left[H(\phi)\sin\phi - E(\phi)\cos(\phi)\right] - \frac{V}{\Omega r}\left[H(\phi)\cos\phi + E(\phi)\sin(\phi)\right]$$

$$H(\phi) = \sin(\phi) + \tfrac{1}{4}\sigma C_d(\phi)$$

$$E(\phi) = \tfrac{1}{4}\sigma C_{l,\alpha}\left(\theta^* - \phi\right)$$

From the physics of the problem it is known that $g(0)$ and $g(\pi/2)$ have opposite signs such that $g(0) * g(\pi/2) < 0$ even if we don't know the sign of $g(0)$, since we know that $0 < \phi < \pi/2$. Therefore, let us now define that $\phi_{left} = 0$ and $\phi_{right} = \pi/2$ are the two boundary values of the range where the solution is located. Obviously the values of $g(\phi)$ at those boundaries are $g_{left} = g(0)$ and $g_{right} = g(\pi/2)$ respectively. Therefore, the solution is bounded within the range of $\phi_{left} < \phi < \phi_{right}$ with the initial conditions that $\phi_{left} = 0$, $g_{left} = g(0)$ and $\phi_{right} = \pi/2$, $g_{right} = g(\pi/2)$.

An approximate value of the solution is then calculated using the regula falsi method as follows

$$\phi_{new} = \phi_{left} - \frac{\triangle\phi}{\triangle f}.f_{left}$$

$$\triangle\phi = \phi_{right} - \phi_{left}$$

$$\triangle f = f_{right} - f_{left}$$

The new values of $C_d(\phi_{new}), H(\phi_{new}), E(\phi_{new})$ are then calculated and utilized to calculate the value of $g_{new} = g(\phi_{new})$. If $g_{new} = 0$ then we have found the solution. If $|g_{new}| \le \varepsilon$ where ε is a predetermined small positive number, then we may also conclude that ϕ_{new} is the solution we are looking for, or $\phi_{sol} = \phi_{new}$. If the solution hasn't been found then we need to do further calculations.

One of the boundary points is now shifted as follows

If $g_{new} * g_{left} < 0$ then replace $(\phi_{right}, g_{right})$ by (ϕ_{new}, g_{new}) since obviously g_{new} and g_{left} have opposite signs.

If $g_{new} * g_{left} > 0$ then replace $\left(\phi_{left}, g_{left}\right)$ by $\left(\phi_{new}, g_{new}\right)$ since obviously g_{new} and g_{left} have the same signs.

The regula falsi method is then used to compute the new value of ϕ_{new} and the whole procedure is repeated until convergence is achieved. One cycle of computations for the new value of ϕ_{new} is called an iteration. To prevent the possibility that the procedure doesn't converge and the computer program goes into an infinite loop, it is always prudent to specify the maximum number of iterations that the computer program is allowed to perform before it is stopped. During the debugging process of writing the computer program, it is always possible that an unexpected typing or other programming mistakes may be made which render the program to be doing computations that are not intended by the programmer.

In the modified regula falsi method, we need to include the following refinement to the procedure described above.

After the first computation of ϕ_{new} and g_{new}, if g_{new} has the same sign as g_{left}, then $\left(\phi_{left}, g_{left}\right)$ is replaced by $\left(\phi_{new}, g_{new}\right)$ and $\left(\phi_{right}, g_{right}\right)$ is replaced by $\left(\phi_{right}, \tfrac{1}{2} g_{right}\right)$. On the other hand if g_{new} has the same sign as g_{right} then $\left(\phi_{right}, g_{right}\right)$ is replaced by $\left(\phi_{new}, g_{new}\right)$ and $\left(\phi_{left}, g_{left}\right)$ is replaced by $\left(\phi_{leftt}, \tfrac{1}{2} g_{leftt}\right)$.

Following that, if a boundary point is stagnant twice in a row then the stagnant boundary point $\left(\phi_{stag}, g_{stag}\right)$ should be replaced by $\left(\phi_{stag}, \tfrac{1}{2} g_{stag}\right)$.

After the value of ϕ at radial station *r* has been calculated, the values of the thrust grading and torque grading can be calculated as follows

$$V_R = \frac{V}{F(\phi)} \qquad \text{or} \qquad V_R = \frac{\Omega r}{G(\phi)}$$

$$\frac{dT}{dr} = \tfrac{1}{2} Bc\rho V_R^2(\phi)\left[C_{l,\alpha}\alpha.\cos\phi - C_d(\alpha)\sin\phi\right]$$

$$\frac{dQ}{dr} = \tfrac{1}{2} Bc\rho V_R^2(\phi)\left[C_{l,\alpha}\alpha.\sin\phi + C_d(\alpha)\cos\phi\right].r$$

where $\alpha = \theta^* - \phi$

It should be noted that the calculation of the thrust and torque gradings doesn't need to involve the calculation of the inflow and swirl factors explicitly. The factors *a* and *b* are involved implicitly in the calculation of the inflow angle ϕ. As it turns out, the expression for the function $g(\phi)$ as discussed above is valid even when the propeller is stationary or when $V = 0$. Therefore, a single computer program can be used to compute the performance of a stationary or a moving propeller at any given speed.

An algorithm for the computation of a propeller's performance is given in the next section.

6 PROPELLER PERFORMANCE COMPUTATIONAL ALGORITHM

The problem we wish to solve may be posed as follows.

Given the geometry of the propeller blades, it should be possible to estimate reasonably accurately the performance of the propeller at any setting of the operating conditions. The propeller is assumed attached to an aircraft that is cruising at a certain altitude at a certain speed, or the aircraft may be stationary with the engine being at full power waiting to take-off. It is assumed that the atmosphere is satisfactorily approximated by the International Standard Atmosphere, so that the air density, temperature and pressure can be calculated if the flight altitude is given.

The forward speed of the propeller as well as its rotational speed must be given.

The number of propeller blades is given and the blades are attached symmetrically around the hub. The radius of the propeller, Rtip, is given, as is the hub radius, Rhub.

The hub radius is in many cases around 20 percent of the tip radius. The attachment of the blade to the hub is such that the whole blade can be rotated about an axis perpendicular to the hub axis, which is known as the blade pitch axis. The geometry of the blade is given as the geometry of a number of blade elements that are obtained if the blade is "cut up" using planes perpendicular to the blade pitch axis. Every blade element has the same width, which is given by (Rtip – Rhub) / Nbel, where Nbel is the number of blade elements making up the whole blade. A blade element may be considered as being a small part of a 2-dimensional infinitely long aerofoil. The shape of an aerofoil is defined by the coordinates of a large but finite number of points on its surface. The straight line joining the nose (forward most point) to the tail (rearward most point) of the aerofoil is known as its chord line. The local aerofoil pitch angle, θ, is the angle subtended by the aerofoil's chord line and the plane of the propeller disc, which is perpendicular to the hub axis. The local pitch angle varies widely along the radial distance, due to the relatively severe geometric twist of the blade. It is possible that the pitch angle at the blade root may be close to 90 degrees while the pitch angle at the propeller tip may be closer to zero degree. The overall blade pitch angle is often defined to be the local pitch angle of the aerofoil at a radial station of 70 percent propeller radius. The geometry of the propeller blade is defined for a particular value of blade pitch, for example for a blade pitch of zero degree. If the whole blade is then turned around through an angle of β about its pitch axis, then the geometry of all elemental aerofoils must be turned through the same angle about the pitch axis, which may be along the leading edge of the blade or very close and parallel to it. It should be noted that a modern propeller blade usually has a tip region that is sweptback to delay the onset of shock wave, since the vector sum of the rotational speed and the forward speed of the tip region of the blade is certainly quite close to the sonic speed. Therefore, the tip region of a propeller blade should be treated rather carefully. The rotational speed of the blade element there should be multiplied by the cosine of the sweep angle, since the relevant speed is that which is perpendicular to the leading edge of the aerofoil. The sweep angle is the angle between the blade leading edge and the radial direction or the direction of the pitch axis.

The aerodynamic performance of a blade element is assumed to be the same as that of the infinitely long 2-dimensional aerofoil that it is a part of. It is obvious that this is not strictly speaking true. However, without this assumption the problem becomes extremely complicated since we have to deal with a complex 3-dimensional in a rotating coordinate system of reference. It should also be remembered that the BEMT method attempts to address the 3-dimensional effect by introducing the concepts of inflow and swirl factors.

Since the geometry of each aerofoil section is given, therefore we can use CFD computer softwares based on the compressible, viscous Navier-Stokes equation of motion to numerically compute the aerofoil aerodynamic properties. If it is at all possible, then a check should be made by conducting measurements in a wind tunnel to confirm the results of computation. It is assumed here that the range of operating angle of attack of each aerofoil is sufficiently small, well below the stall angle so that the flow around the aerofoil is always attached and is never separated. With that restriction, it is reasonable to assume that the lift coefficient of the aerofoil varies linearly with angle of attack, α, as follows.

$$C_l = C_{l,\alpha}.(\alpha - \alpha_0)$$

where $C_{l,\alpha}$ is the lift curve slope, α_0 is the zero lift angle of attack and α is the flow angle of attack.

As discussed in the previous sections the angle of attack is related to the local pitch angle, θ, and inflow angle, ϕ, as follows

$$\alpha = \beta + \theta - \phi$$

Therefore, the lift coefficient can be written as

$$C_l = C_{l,\alpha}.(\beta + \theta - \alpha_0 - \phi) = C_{l,\alpha}.(\theta^* - \phi)$$

It is assumed that the value of the lift curve slope $C_{l,\alpha}$ of each aerofoil section is known, either as a result of wind tunnel measurement or from CFD numerical simulation. It should be noted that the slope of the lift curve of any aerofoil is close to but less than 2π.

It is further assumed that the drag coefficient, C_d, of each aerofoil section is a constant, independent of angle of attack, and is obtainable from CFD simulation or from measurement in a wind tunnel. It is also to be noted that it has a small value around 0.02 or 0.03.

With the above constraints in mind, the algorithm for the computation of a propeller performance can now be described below.

1. The initial data are given

Epsilon is a small positive number, say 0.000001.

Bnum is the number of blades of the propeller.

If given in degrees must convert to radian, i.e. multiplied by $\pi/180$

Nbel is the number of blade elements making up the whole propeller blade.

Rtip is the radius of the propeller in *m*

Rhub is the hub radius in *m*

Compute width of each blade element Delw in *m*

Delw = (Rtip – Rhub)/ Nbel in *m*

For each value of j, from j = 1 to j = Nbel, the following data are given

SlopeCl(j) is the lift curve slope for the j^{th} aerofoil in unit of rad^{-1}

Cdrag(j) is the drag coefficient of the j^{th} aerofoil

Alfa0(j) is the zero lift angle of attack of the j^{th} aerofoil in *rad*

Theta(j) is the local pitch angle of the j^{th} aerofoil in *rad*

Chord(j) is the chord length of the j^{th} aerofoil in *m*

For each value of j, from j = 1 to j = Nbel, the following data must be computed

Radial distance of each blade element radial station, Rbel(j), in *m*

Rbel(j) = Rhub + (j-1) x Delw

Blade element solidity at each blade element radial station, Sigma(j)

Sigma(j) = Bnum x Chord(j) / (2π x Rbel(j))

SigCd(j) is the variable $\frac{1}{4}\sigma.C_d$

SigCd(j) = 0.25 x Sigma(j) x Cdrag(j)

SigCl(j) is the variable $\frac{1}{4}\sigma.C_{l,\alpha}$

SigCd(j) = 0.25 x Sigma(j) x SlopeCl(j)

1.1 The following initial data must also be given

Beta is the blade pitch angle in *rad*

Rho is the air density at flight altitude in kg/m^3

Vfor is the forward speed of the cruising aircraft in m/s

Nrot is the rotational speed of the propeller in *rpm* (rotations per minute)

Calculate rotational speed in radian per second

Omega = Nrot x π / 30 *rad/s*

For each value of j, from j = 1 to j = Nbel, the following data must also be computed

Vrat(j) is the ratio of forward to rotational velocities at the j^{th} radial station

Vrat(j) = Vfor / (Omega x Rbel(j))

TtStar(j) is the variable $\theta^* = \beta + \theta - \alpha_0$

TtStar(j) = Beta + Theta(j) – Alfa0(j)

2.For each value of j, from j = 1 to j = Nbel compute the thrust grading, dTdr(j), and the torque grading, dQdr(j). This requires that the value of the inflow angle ϕ(j) must be computed using a numerical method. Here we choose the modified regula falsi method to compute ϕ(j).

2.1 Initialize the solution boundaries as well as initial guess for the sought for solution

Phileft = 0

Phirite = π/2

Gleft = - (SigCl(j) x TtStar(j) + Vrat(j) x SigCd(j))

Grite = 1 + SigCd(j) + Vrat x SigCl(j) x (π/2 – TtStar(j))

Delphi = Phirite – Phileft

Delg = Grite – Gleft

Phinew = Phileft – (Delphi/Delg) x Gleft

Sinfi = sin (Phinew)

Cosfi = cos (Phinew)

Hphi = Sinfi + SigCd(j)

Ephi = SigCl(j) x (Ttstar(j) – Phinew)

Fsinfi = Hphi x Sinfi –Ephi x Cosfi

Gnew = Fsinfi – Vrat(j) x (Hphi x Cosfi + Ephi x Sinfi)

Islb = 1

Isrb = 1

2.2 Do the following set of computations (iteration) for a maximum allowable number of iterations, Itermax, unless solution has converged before that.

Check to see which boundary point must be shifted. Also check if the same boundary point has been shifted twice in a row, in which case the function G value of the stagnant point should be halved.

No. 1 If Gleft x Gnew is negative, then do the following

Phirite = Phinew

Grite = Gnew

Isrb = 0

No. 2 If Islb = 1, then do the following

Gleft = $\frac{1}{2}$ Gleft

Else (if Islb not equal 1) do the following

Islb = 1

End No. 2 if operations

Else (if Gleft x Grite is not negative) then do the following

Phileft = Phinew

Gleft = gnew

Islb = 0

No. 3 If Isrb = 1, then do the following

Grite = $\frac{1}{2}$ Grite

Else (if Isrb not equal 1) do the following

Isrb = 1

End of No. 3 if operations

End of No.1 if operations

Now compute a new improved approximation of the sought for solution

Delphi = Phirite – Phileft

Delg = Grite – Gleft

Phinew = Phileft – (Delphi/Delg) x Gleft

Sinfi = sin (Phinew)

Cosfi = cos (Phinew)

Hphi = Sinfi + SigCd(j)

Ephi = SigCl(j) x (Ttstar(j) – Phinew)

Fsinfi = Hphi x Sinfi –Ephi x Cosfi

Gnew = Fsinfi – Vrat(j) x (Hphi x Cosfi + Ephi x Sinfi)

Now check to see if the solution has converged.

If the absolute value of Gnew is equal to or less than epsilon, then we have found the solution. Jump out of the iteration loop because solution has converged. Jump to procedure number 2.4

If the absolute value of Gnew is still too big (far from zero), and the number of iterations that have been performed is less than Itermax then return to 2.2. On the other hand if the number of iterations is already equal to Itermax then the program has failed, the solution diverges and jump to 2.3

2.3 The solution has diverged. Stop the program and tell the user that program has failed.

2.4 The solution has been found

Phisol(j) = Phinew

2.5 Now calculate the resultant velocity of the air flow when passing through the

propeller disc, Vres

Vres = Vfor x Sinfi / Fsinfi

2.6 Now calculate the thrust and torque gradings

Bcro = 0.5 x Bnum x Chord(j) x Rho x $Vres^2$

Cl = SlopeCl(j) x (TtStar(j) – Phisol(j))

DTdr(j) = Bcro x (Cl x Cosfi – Cdrag(j) x Sinfi)

DQdr(j) = Bcro x (Cl x Sinfi + Cdrag(j) x Cosfi) x Rbel(j)

Now go back to procedure number 2 for the next value of j.

If j is already equal to Nbel, then go to procedure number 3

3. Now calculate the thrust and torque of the propeller using the trapezoidal rule of integration

Thrust = $\frac{1}{2}$ x dTdr(1)

Torque = $\frac{1}{2}$ x dQdr(1)

Now do the following for each successive value of j until j = Nbel

Thrust = Thrust + dTdr(j)

Torque = Torque + dQdr(j)

After j has reached the value of Nbel, then do the following

Thrust = Thrust x Delw

Torque = Torque x Delw

4. Now calculate the theoretical efficiency of the propeller

Efficiency = ((Thrust x Vfor) / (Torque x Omega)) x 100 percent

5. The program has finished successfully.

To calculate thrust, torque and efficiency of the propeller for a different value of blade pitch go back to procedure number 1.1 and change the value of Beta.

The same comment applies if the dependence of propeller performance on forward speed, or rotational speed or flight altitude is required.

7 CONCLUSION

In this report we have discussed how the performance of a propeller can be predicted numerically by using the BEMT (blade element momentum theory) method.

This method is based on the momentum theory of propulsion where the propeller is replaced simply by an actuator disc that creates a sudden jump in air pressure as the airflow passes through the plane of the propeller disc.

The momentum theory is useful in getting some understanding on how the efficiency of a propeller depends on the additional airspeed that is induced by the propeller. The thrust that is produced by a propeller is merely the reaction acting on the propeller to the change in momentum that the propeller imparts on the air flowing through it. The rate of momentum change is the product of mass flow rate multiplied by the change of velocity of the airflow due to being induced by the propeller. Therefore, a large thrust can be obtained by moving a large mass of air and imparting to it a small change of velocity. On the other hand, the same amount of thrust can also be achieved by moving a smaller mass of air and imparting a larger velocity change to it. The momentum theory is capable of predicting that it is more efficient to move a larger mass of air and imparting only a small velocity change to it. This is the fundamental physical reason why a higher bypass fan ratio of a fan jet engine is more efficient than an engine with a smaller bypass ratio, and that for slower aircraft a turboprop engine is more efficient than a jet engine.

While, the momentum theory is quite useful for theoretical study purposes it is not very useful as a design tool for designing a propeller. However, when combined with the aerofoil or blade element theory in the blade element momentum theory, the method becomes very useful as a tool for the analysis and design of propellers in general.

The BEMT method borrows some concepts from the lifting line theory in aircraft wing analysis and design. The highly complex 3-dimensional fluid flow over an aircraft wing is considerably simplified by assuming that the flow over a small element of the wing may be considered to behave like a 2-dimensional flow, not interacting with the flows over the neighbouring wing elements. This is of course contrary to the actual flow over a wing that can be observed, where there is a strong cross flow along the span of the wing, particularly for the flow over a small aspect ratio wing. This 3-dimensional effect is taken into account in the Lifting Line Theory by modeling the flow over the wing, not just as the flow over a lifting vortex bound on the span of the wing, but in addition a vortex sheet is also shed at the wing trailing edge. The trailing vortex sheet induces a downward velocity component to the incoming airflow, thus effectively reduces the angle of attack seen by the wing. This in turn has the effect of reducing the lift produced by the wing, and it also creates a lift dependent drag or induced drag, which doesn't exist for a purely 2-dimensional flow.

In BEMT method, the propeller blade is also assumed to consist of a large number of blade elements, each of which behaves like a 2-dimensional aerofoil. The flow over each blade element is assumed to behave like a 2-dimensional flow, not interacting with the flow over the other adjacent blade elements. The 3-dimensional effect is then modeled back into the propeller flow by assuming that a vortex sheet is shed at the blade's trailing edge. This vortex sheet then induces both an axial as well as an azimuthal velocity component to the incoming flow seen by the propeller blade. Due to the rotating nature of the propeller flow, it is very difficult to analyze directly the effect of the helical vortex sheet on the flow, e.g. by means of the fundamental Biot-Savart equation for vortex flow, as is done in the case of lifting line theory for wing flow analysis.

However, it is possible to apply the momentum theory on the annulus formed by the rotating blade element, and combining it with the aerofoil theory then the induced axial and azimuthal velocity components can be shown to be dependent only on a single parameter, namely the inflow angle. Furthermore, the BEMT method can be utilized to calculate the inflow angle provided that the 2-dimensional aerodynamic properties of the blade elements are known from the 2-dimensional aerofoil theory. The aerodynamic properties of 2-dimensional aerofoils are much easier to measure or simulate numerically, compared to a rotating 3-dimensional flow, which is the actual flow for airflow around a real propeller.

In this report, the theoretical foundation for the blade element momentum theory is discussed in great detail, so that it is sufficient to be translated into an algorithm for solving this type of problem.

The algorithm is suitable for analyzing the propeller performance of any propeller if the geometry of the propeller blade is given. The performance of a propeller obviously depends on the operating conditions, such as the forward speed of the aircraft, the rotational speed of the blades, and the density of the fluid as well as the blade pitch setting angle. If all those data are given, then the algorithm described in this report is capable of predicting the performance of a propeller, provided that the aerodynamic properties of each blade element making up the overall blade are also given.

8 RECOMMENDATIONS

In this report we have discussed the possibility of using the Blade Elemen Momentum Method to calculate the performance of any arbitrary propeller.

The method is dependent on the availability of the aerodynamic properties of the blade elements making up the whole blade.

Such data can be calculated using any CFD computer software, preferrably one that is based on solving the Reynolds Averaged Navier-Stokes equation of motion. The accuracy of the computational results is very much dependent on many factors, such as the choice of shape, size, number of points making up the computational mesh as well as how those mesh or grid points are distributed within the solution field. The accuracy of the solution is also heavily dependent on the choice of the turbulence model selected for the computations.

It is recommended, therefore, that a study be conducted and a report produced on how to effectively, efficiently and correctly utilize the chosen CFD software. A rather popular general purpose CFD software, which solves the RANS equation of motion, is FLUENT. It is recommended, therefore, that the proper usage of FLUENT should be studied.

After a reasonable mastery of the usage of CFD softwares such as FLUENT has been achieved, then a known propeller's performance should be investigated based on its geometry and operating conditions using the BEMT technique. Given the geometry of the blade elements of the propeller's blade, then their aerodynamic properties should be computed using FLUENT and the results are fed into the BEMT program to be used as data for calculating the propeller's performance for different operating conditions. If at all possible the computed propeller's performance should be compared to the actual perfomance of the propeller, to check on the accuracy of the BEMT method.

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BEMT Algorithm for the Prediction of the Performance of Arbitrary Propellers

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6. Abstract
This report describes details of the Blade Element Momentum Theory (BEMT) and its application for the calculation of the performance of any arbitrarily shaped propeller.

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