RF Coils, Helical Resonators and Voltage Magnification by Coherent Spatial Modes

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"Is there, I ask, can there be, a more interesting study than that of alternating currents."
Nikola Tesla, (Life Fellow, and 1892 Vice President of the AIEE)[1]

Abstract – ***By modeling a wire-wound coil as an anisotropically conducting cylindrical boundary, one may start from Maxwell's equations and deduce the structure's resonant behavior. Not only can the propagation factor and characteristic impedance be determined for such a helically disposed surface waveguide, but also its resonances, "self-capacitance" (so-called), and its voltage magnification by standing waves. Further, the Tesla coil passes to a conventional lumped element inductor as the helix is electrically shortened.***

Keywords – coil, helix, surface wave, resonator, inductor, Tesla.

I. INTRODUCTION

One of the more significant challenges in electrical science is that of constricting a great deal of insulated conductor into a compact spatial volume and determining the resulting structure's electrical properties. The problem is not only of interest at very low frequencies, where the arrangement can be treated as a lumped element (an inductor), but also at frequencies where the current distribution over the structure is not uniform and the conventional lumped element assumption fails. A simple helical coil geometry is considered in the following note. (It is a common misconception that a full field analysis is necessary only at relatively high frequencies. In point of fact, it is *essential* whenever the current distribution over the structure is not uniform.)

Adler, Chu and Fano have pointed out that there are three important characteristics of high Q linear systems: (1) their free oscillations are slightly damped; (2) Their input impedance has a rapid and radical variation around resonance; (3) Their field distributions possess characteristic *space* patterns that accompany the sharply selective behavior of the system. It is this third feature that is often "…overlooked in lumped networks and is of great importance in understanding distributed resonant systems."[2] A helically wound Tesla coil is just such a system, as will be shown in the following note. (The analysis is performed during the linear system – pre-discharge epoch, while the voltage rise phenomenon is occurring. The system has a nonlinear load during discharge, of course.)

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II. CYLINDRICAL HELICES

A. Problem Formulation

A uniform helix is described by its radius (r = a), its pitch (or turn-to-turn wire spacing "s"), and its pitch angle ψ, which is the angle that the tangent to the helix makes with a plane perpendicular to the axis of the structure (z). Geometrically, $\psi = \cot^{-1}(2\pi a/s)$. The wave equation is not separable in helical coordinates and there exists no rigorous solution of Maxwell's equations for the solenoidal helix.[3] However, at radio frequencies a wire-wound helix with many turns per free-space wavelength (e.g., a Tesla coil) may be modeled as an idealized anisotropically conducting cylindrical surface that conducts only in the helical direction. The conductivity normal to the helical path is taken to be zero. (This cylindrical tube-shaped structure is the classic *sheath helix* model due to Ollendorf.[4]) Formally, for time-harmonic fields (assume $e^{j\omega t}$ time variation), the homogeneous vector Helmholtz equations

$$\nabla^2 \begin{Bmatrix} \vec{E} \\ \vec{H} \end{Bmatrix} + \boldsymbol{k}^2 \begin{Bmatrix} \vec{E} \\ \vec{H} \end{Bmatrix} = 0 \qquad (\boldsymbol{for} \quad \boldsymbol{r} \neq \boldsymbol{a}) \qquad \textbf{(1)}$$

where $k = \omega(\mu\varepsilon)^{1/2} = 2\pi/\lambda = k_o(\varepsilon_r\mu_r)$, must be solved subject to a set of appropriate boundary conditions. [The passage to quasistatic field theory and lumped circuit analysis occurs as the wavelength becomes infinite (or, equivalently, as $\boldsymbol{c} \to \infty$).] The Helmholtz equation is separable in eleven three-dimensional orthogonal coordinate systems so that product solutions may be formed. The helical coordinate system is not one of them.

The helical structure supports propagation along the longitudinal (z) axis with traveling wave variations of the form $e^{j(\omega t - \beta z)}$, where β is to be determined. (We have temporarily neglected dissipation so that the propagation factor $\gamma = \alpha + j\beta \to j\beta$.) The fields may be decomposed into transverse and longitudinal components, there being *both* TE *and* TM modes present along this anisotropic wave guide. Further, the ∇ operator separates into a transverse and a longitudinal operator, that is, $\nabla = \nabla_t - (\boldsymbol{j\beta})\bar{z}$, to give the Laplacian operator $\nabla^2 = \nabla_t^2 - \beta^2$, so that the D'Alembertian operator, $\left[\nabla^2 + \boldsymbol{k}^2\right] = \left[\nabla_t^2 + \left(\boldsymbol{k}^2 - \boldsymbol{\beta}^2\right)\right]$, and Equation (1) separates into the operations:

$$\nabla_t^2 \begin{Bmatrix} \bar{E}_t \\ \bar{H}_t \end{Bmatrix} - \tau^2 \begin{Bmatrix} \bar{E}_t \\ \bar{H}_t \end{Bmatrix} = 0 \quad \textbf{(2)}$$

on the transverse field components, and the operations

$$\nabla_t^2 \begin{Bmatrix} E_z \\ H_z \end{Bmatrix} - \tau^2 \begin{Bmatrix} E_z \\ H_z \end{Bmatrix} = 0 \quad \textbf{(3)}$$

on the longitudinal field components. In these expressions, the all important radial wave number, τ, is given by

$$\tau^2 = -\left(\gamma^2 + k^2\right) = \beta^2 - k^2 \ . \quad \textbf{(4)}$$

The central issue for wave propagation on helical structures is the behavior of τ as the frequency and helix geometry (a, s, and ψ) are varied.

Unlike the case of a TEM transmission line, and this is important, $E_z \neq 0$ and $H_z \neq 0$ on the helix wave guide. Furthermore, $\beta \neq k$ (where $k^2 = \omega^2\mu\varepsilon$ is the plane wave propagation constant). From the cylindrical symmetry, the axial component of the internal (i) and external (o) time-harmonic fields will each be a superposition of forward propagating modes in the form of a product of radial, azimuthal, and axial functions of the form

$$E_z^{i,o}(r,\phi,z) = \left[A_n^{i,o} I_n(\tau_n r) + B_n^{i,o} K_n(\tau_n r)\right] e^{j(\omega t - n\phi - \beta_n z)} \quad \textbf{(5)}$$

$$H_z^{i,o}(r,\phi,z) = \left[C_n^{i,o} I_n(\tau_n r) + D_n^{i,o} K_n(\tau_n r)\right] e^{j(\omega t - n\phi - \beta_n z)} \quad \textbf{(6)}$$

where the eight constants A, B, C, D [both inside (superscript i) and outside (superscript o)] are to be determined for each mode from the boundary conditions and the driving source, and β_n is the propagation constant along the helix for the n^{th} mode. There will also be a set of linearly superposed backward propagating modes, too, but, for the time being, we will consider only the forward propagating modes. The radial functions $I_n(x)$ and $K_n(x)$ are the modified Bessel functions of the first and second kind, respectively, of order n and argument x. For the case of interest, we have circular symmetry azimuthally around the solenoid, so we take only the mode with n = 0.[5,6] (This has been called the helix transmission line T_O mode.[7])

B. Physical Constraints

In order to find the other field components inside and outside of the helix, one employs Ampere's law and Faraday's law to solve for E_r, E_φ, H_r, and H_φ in terms of derivatives of E_z and H_z. Consider the n = 0 mode (azimuthal symmetry in the fields). Physically, the fields must be finite at r = 0 and vanish as $r \to \infty$. From the properties of the modified Bessel functions, this implies that A^o, B^i, C^o, D^i must all vanish. Using the modified Bessel function derivative relations

$$\frac{\partial I_0(\tau r)}{\partial r} = \tau I_1(\tau r) \quad and \quad \frac{\partial K_0(\tau r)}{\partial r} = -\tau K_1(\tau r) \quad \textbf{(7)}$$

and the fact that $K_0(\tau r)$ diverges as $r \to 0$, expressions for the fields within the helix $(r \le a)$ can be found. Outside the helix $(r \ge a)$, the $I_0(\tau r)$ diverges as $r \to \infty$, and expressions for the external fields are readily determined. However, these expressions are all in terms of the unknown constants A^i, B^o, C^i, D^o and the propagation constant τ. These constants and the propagation factor can be determined by imposing the anisotropic boundary conditions across the helical surface at r = a.

C. Sheath Helix Boundary Conditions

There are three boundary conditions to be considered at the surface of the anisotropic sheath helix. One may summarize these as follows:

(1) The Maxwell boundary conditions at the cylindrical surface make the internal and external components of field strength along (i.e., parallel to) the wire vanish as $r \to a$: **E**(a) = 0. This may be written as

$$\bar{E}_{\parallel}^{i}(a^-) = \bar{E}_{\parallel}^{o}(a^+) = 0 \quad \textbf{(8)}$$

(2) The components of the electric field strength normal to the wire are continuous across the cylindrical boundary as approached from within or without:

$$\bar{E}_{\perp}^{i}(a^-) = \bar{E}_{\perp}^{o}(a^+) \quad \textbf{(9)}$$

(3) Finally, the components of the magnetic field strength along the wire are continuous across the cylindrical boundary:

$$\bar{H}_{\parallel}^{i}(a^-) = \bar{H}_{\parallel}^{o}(a^+) \ . \quad \textbf{(10)}$$

Imposing these boundary conditions on the expressions for the internal and external fields gives four equations in the four remaining unknown constants A^i, B^o, C^i, D^o and the propagation constant τ.

$$A^i I_0(\tau a)\sin\psi - \frac{j\omega\mu}{\tau} C^i\, I_1(\tau a)\cos\psi = 0 \quad \textbf{(11)}$$

$$B^o K_0(\tau a)\sin\psi + \frac{j\omega\mu}{\tau} D^o\, K_1(\tau a)\cos\psi = 0 \quad \textbf{(12)}$$

$$\begin{aligned} &A^i I_0(\tau a)\cos\psi + \frac{j\omega\mu}{\tau} C^i\, I_1(\tau a)\sin\psi \\ &= B^o K_0(\tau a)\cos\psi - \frac{j\omega\mu}{\tau} D^o\, K_1(\tau a)\sin\psi \end{aligned} \quad \textbf{(13)}$$

$$\begin{aligned} &C^i I_0(\tau a)\sin\psi + \frac{j\omega\mu}{\tau} A^i\, I_1(\tau a)\sin\psi \\ &= D^o K_0(\tau a)\sin\psi - \frac{j\omega\varepsilon}{\tau} B^o\, K_1(\tau a)\cos\psi \end{aligned} \quad \textbf{(14)}$$

These are four linear equations in five unknowns and the solutions may be found either by direct algebraic substitution or by the method of determinants.

D. Field Distributions and Propagating Modes

Simultaneous solution of the first three boundary condition equations (for the n = 0 case) permits one to express all the constants (B^o, C^i, D^o) in terms of A_0^i. Using these constants and the field expressions determined for $(r \le a)$ and $(r \ge a)$ in paragraph B, one has the fields inside and outside of the helical structure all in terms of the constant A^i (which can be directly related to the source driving function). The result is the following set of forward propagating TE and TM fields.

(For $r \le a$)

$$E_z^i = A_0^i\, I_0(\tau r)\, e^{j(\omega t - \beta z)} \tag{15}$$

$$E_r^i = \frac{j\beta}{\tau} A_0^i\, I_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{16}$$

$$E_\phi^i = -\frac{I_0(\tau a)}{I_1(\tau a)} \tan\psi\, A_0^i\, I_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{17}$$

$$H_z^i = \frac{\tau}{j\omega\mu} \frac{I_0(\tau a)}{I_1(\tau a)} \tan\psi\, A_0^i\, I_0(\tau r)\, e^{j(\omega t - \beta z)} \tag{18}$$

$$H_r^i = \frac{\beta}{\omega\mu} \frac{I_0(\tau a)}{I_1(\tau a)} \tan\psi\, A_0^i\, I_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{19}$$

$$H_\phi^i = \frac{j\omega\varepsilon_0}{\tau} A_0^i\, I_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{20}$$

(For $r \ge a$)

$$E_z^o = \frac{I_0(\tau a)}{K_0(\tau a)} A_0^i\, K_0(\tau r)\, e^{j(\omega t - \beta z)} \tag{21}$$

$$E_r^o = -\frac{j\beta}{\tau} \frac{I_0(\tau a)}{K_0(\tau a)} A_0^i\, K_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{22}$$

$$E_\phi^o = -\frac{I_0(\tau a)}{K_1(\tau a)} \tan\psi\, A_0^i\, K_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{23}$$

$$H_z^o = -\frac{\tau}{j\omega\mu} \frac{I_0(\tau a)}{K_1(\tau a)} \tan\psi\, A_0^i\, K_0(\tau r)\, e^{j(\omega t - \beta z)} \tag{24}$$

$$H_r^o = \frac{\beta}{\omega\mu} \frac{I_0(\tau a)}{K_1(\tau a)} \tan\psi\, A_0^i\, K_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{25}$$

$$H_\phi^o = -\frac{j\omega\varepsilon_0}{\tau} \frac{I_0(\tau a)}{K_0(\tau a)} A_0^i\, K_1(\tau r)\, e^{j(\omega t - \beta z)} \tag{26}$$

While they might appear formidable at first blush, this set of field expressions is an extremely useful tool for determining the properties of both lumped and distributed RF coils. The asymptotic behavior of the $K_n(\tau r)$ is exponential, which implies that the helix fields are actually surface waves guided by the helix.[8] In passing, one should also note that, both internally and externally, the ratio of the longitudinal to azimuthal field components is given by the expression

$$\frac{E_z}{E_\phi} = \cot\psi = \frac{2\pi a}{s} \tag{27}$$

so the field ratio is the ratio of the circumference to the helix pitch, E_Z » E_φ. This is the basis, in Maxwell's equations, for the assertion made long ago by Contaxes and Hatch that this field ratio is a basic geometrical property of coils.[9] (They had based their arguments upon inductor formulae which presuppose that the velocity of propagation on the helix is infinite, i.e. - there are no standing waves on the coils.) The same results were obtained in the quasi-static approximation by Fano, Chu and Adler.[10]

Equations (15)-(26) for the propagating waves may be plotted to show the propagating field structure about the helix.[11,12] Remember that the helix has been assumed to be infinitely long, so there are only forward traveling waves at present. Later, with the introduction of an arbitrary load impedance, reflected waves will occur and a standing wave distribution will arise, as with any wave guiding system.

E. Propagation Constant

Equation (14) for the anisotropic boundary conditions remains and has not yet been used. After simultaneously solving Equations (11)-(13) for the constants (B^o, C^i, D^o) in terms of A_0^i, one may substitute these directly into Equation (14) to get the eigenvalue equation for τ:

$$(ka)^2 \frac{K_1(\tau a)\, I_1(\tau a)}{K_0(\tau a)\, I_0(\tau a)} = (\tau a)^2 \tan^2\psi\,. \tag{28}$$

This is an extremely important equation, and it must be solved for τ when the frequency, ω, and the helix parameters "a" and "s" (and the angle ψ) are specified. Since this is the characteristic determinant of Equations (11)-(14), it is called the determinantal equation for τ.

A trivial set of solutions occurs when $\tau^2 = 0$. From Equation (4), these are TEM waves with $\beta = \pm k$. However, for slow waves on the helix, τa >> ka (because $\lambda_g << \lambda_o$). It should be recalled that we have been treating only the cylindrically symmetric (n = 0) case. At ultra-high and super high frequencies, the fields will no longer retain their circumferential symmetry and higher order modes will begin to become of interest. The determinantal equation for these modes can readily be obtained, and is given in the literature.

The transcendental expression given by Equation (28) is readily solved numerically for a given geometry. When

$\tau a \geq 10$, which occurs in the regime where either the circumference of the helix is greater than $\lambda_o/4$ or there are *not* a huge number of turns per wavelength (as in traveling wave tubes), the ratio of modified Bessel functions $(K_1 I_1 / K_0 I_0) \to 1$, and $\tau \approx k \csc\psi$. The phase velocity, v_p, in this case becomes

$$\upsilon_p \;=\; \frac{\omega}{\beta} \;\to\; c \sin\psi \,. \tag{29}$$

The earliest work on the helix problem was that of Pocklington[13] (of antenna integral equation fame), whose analysis was in this regime. This expression is *not* very useful when there are a large number of turns per wavelength along the helix or the helix diameter is small in terms of a free-space wavelength (as is the case with Tesla coils). In such cases it is best to actually solve Equation (28) numerically for the propagation constant.

A useful engineering approximation has been found for the fundamental resonance of helices with a large number of turns per wavelength, such as we are presently considering. The separation constant may be expressed in terms of the wave number, k, and the velocity factor, $V_f = v_p/c = k/\beta$, as

$$\tau^2 = -k^2\left(1-\frac{c^2}{\upsilon_p^2}\right) = -k^2\left(1-\frac{1}{V_f^2}\right) \tag{30}$$

where v_p is the phase velocity *along the axis of the helix*, and $k = \omega/c = 2\pi/\lambda_o$. That is,

$$V_f = \frac{\upsilon_p}{c} = \frac{1}{\sqrt{1+\left(\dfrac{\tau}{k}\right)^2}} = \frac{1}{\sqrt{1+\left(\dfrac{M\lambda}{2\pi a}\right)^2}} \tag{31}$$

where τ is to be determined as a root of Equation (28). [The parameter M has been defined as $M = \tau a$.] By plotting the left-hand and right-hand sides of Equation (28) for assumed values of τa, and graphically determining the intersection point, an approximation for M has been determined by Kandoian and Sichak[14] which is appropriate *for quarter-wave resonance* and is valid for helices with $5ND^2/\lambda_o \leq 1$ (where N = 1/s is the number of turns per unit length and D = 2a is the helix diameter), i.e. - for helices with diameters considerably less than a free-space wavelength. Kandoian and Sichak found that M may be represented approximately as $M = 20\pi^2 D^5/(s\lambda)^{2.5}$. In all our publications, we have reexpressed their solution in terms of the velocity factor for propagating waves by the simple formula:

$$V_f = \frac{1}{\sqrt{1+20\left(\dfrac{D}{s}\right)^{2.5}\left(\dfrac{D}{\lambda_o}\right)^{0.5}}} \tag{32}$$

where D = 2a is the helix diameter. We have found that this expression gives *acceptable* results (errors less than 10%) for most practical applications that involve wave propagation on helical resonators with azimuthal field symmetry (i.e., the T_0 transmission line mode). The axial

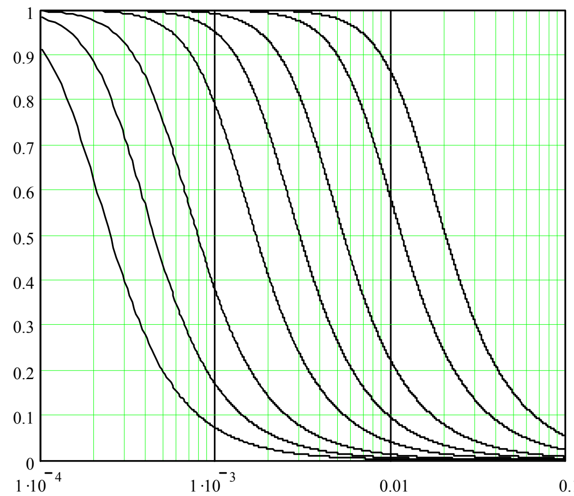

Fig. 1. V_f verses D/λ_o for coils of N = 1/s = 10,000; 5,000; 2,500; 1,000; 500; 250; 100; 50 turns/λ_o, respectively (left to right).

velocity factor has been plotted in Fig. 1 as a function of D/λ_o for a variety of wire spacings. Tightly wound coils are slow wave structures. Experimentally, the wave velocity and velocity factor may be determined by measuring the axial length of standing wave patterns on the helical structure with a movable probe.

F. Helix Characteristic Impedance

There are several expressions that are used in the helix literature as "impedance parameters" (the issue is that there are both TE and TM waves on the anisotropic structure), and we direct our attention to a "Transmission Line Characteristic Impedance" obtained many years ago by Sichak. This relation is particularly useful when considering the helix as a resonator and for working with Smith charts and impedance diagrams.

The derivation proceeds as follows. Considering the helix as a single conductor waveguide, one may determine an equivalent transmission line characteristic impedance in three steps. First, as with a coaxial line, one employs Equation (22) and defines a transverse voltage in some plane z = 0: (Pierce attributes this step to Schelkunoff.[15])

$$\begin{aligned} V_t(0) &= -\int_a^\infty \vec{E}(r,0)\bullet\hat{r}\,dr \\ &= \frac{j\beta}{\tau}\frac{I_0(\tau a)}{K_0(\tau a)}A^i\int_a^\infty K_1(\tau r)\,dr \\ &= \frac{j\beta}{\tau^2}I_0(\tau a)\,A^i \end{aligned} \tag{33}$$

Note that use has been made of the integral relation

$$\int K_1(x)\,dx \;=\; -K_0(x)\,. \tag{34}$$

Then, following Sichak,[16] an effective characteristic impedance may be found from the ratio of the transverse voltage to the conduction current (which is determined from the fields). The longitudinal conduction current follows as:

$$\begin{aligned} I_z &= \oint \vec{H}(a) \bullet d\hat{\ell} \;-\; j\omega\varepsilon \int_0^{2\pi} \int_0^{a} \vec{E} \bullet \hat{n} dA_{CS} \\ &= \int_0^{2\pi} H_\phi^o(a)\, a\, d\phi \;-\; j2\pi\omega\varepsilon \int_0^{a} E_z^o(r)\, r\, dr \\ &= -\left(\frac{jka}{60\tau}\right) A^i \left[\frac{I_0(\tau a)}{K_0(\tau a)} K_1(\tau a) + I_1(\tau a)\right] \end{aligned} \tag{35}$$

where use has been made of the fact that $\omega\varepsilon = k/Z_o$ and the modified Bessel integral relations

$$\int x^n I_{n-1}(x)\, dx \;=\; x^n\, I_n(x) \tag{36}$$

$$\int x^n K_{n-1}(x)\, dx \;=\; -x^n\, K_n(x)\ . \tag{37}$$

The effective "characteristic impedance" for an isolated helical waveguide is then found (just as for a TEM transmission line) from the ratio of the transverse voltage to the longitudinal conduction current, $Z_c = V_t / I_z$, as

$$Z_c = \frac{60}{\tau a} \frac{\beta}{k} \left[\frac{I_0(\tau a)\, K_0(\tau a)}{I_0(\tau a)\, K_1(\tau a) + I_1(\tau a)\, K_0(\tau a)} \right]. \tag{38}$$

Making use of the Bessel Wronskian[17]

$$W\big(I_n(x), K_n(x)\big) = \begin{vmatrix} I_1(x) & K_1(x) \\ I_1'(x) & K_1'(x) \end{vmatrix} \tag{39}$$

which may be expanded using the derivative relations

$$I_1'(x) = I_0(x) - \frac{I_1(x)}{x} \tag{40}$$

$$K_1'(x) = -K_0(x) - \frac{K_1(x)}{x} \tag{41}$$

$$\begin{aligned} W\big(I_n(x), K_n(x)\big) &= -\big[I_0(x)\, K_1(x) + I_1(x)\, K_0(x)\big] \\ &= \frac{-1}{x} \end{aligned} \tag{42}$$

to give the helical wave guide effective characteristic impedance as

$$Z_c = \frac{60}{V_f} I_0(\tau a)\, K_0(\tau a)\ . \tag{43}$$

This important impedance expression is useful for Smith chart and engineering calculations. It is worth noting that, for a helical anisotropic wave guide, the effective characteristic impedance is not merely a function of the geometrical configuration of the conductors (as it would be for lossless TEM coaxial cables and twin-lead transmission lines), but it is also a function of the excitation frequency.

G. Remarks

There are two observations that we wish to make at this point. First, we note that[18]

$$I_n(x) = j^{-n}\, J_n(jx) \tag{44}$$

$$K_n(x) = \frac{\pi}{2} j^{n+1}\, H_n^{(1)}(jx)\ . \tag{45}$$

where the ordinary Bessel function of order n and complex argument and the Hankle function of the first kind of order n and complex argument have been introduced. So, Equation (43) for the characteristic impedance may also be written as

$$Z_c = j \frac{c}{\upsilon} 30\pi\, J_0(jM)\, H_0^{(1)}(jM) \tag{46}$$

which is the expression originally given by Sichak.

Second, we have previously employed the equation

$$Z_c = \frac{60}{V_f} \left[\ell n\left(\frac{4\,h}{D}\right) - 1 \right] \tag{47}$$

as an analytical expression for the helical transmission line characteristic impedance near quarter-wave resonance ($h = \lambda_g / 4$). The axial height of the coil is h, and D is its diameter (both measured in the same units). The formula was actually derived by Schelkunoff as the "average effective characteristic impedance" in his classic transmission line model of the biconical antenna. The only modification was to include slow-wave effects. How does this compare against Equation (43)? The small argument asymptotic expressions for the n = 0 modified Bessel functions of the first and second kind are

$$I_0(x) \;\to\; 1 \tag{48}$$

$$K_0(x) \to -\left(0.577 + \ell n\, \frac{x}{2}\right) = \ell n\left(\frac{1.124}{x}\right)\ . \tag{49}$$

For slow waves ($\beta_g \gg k_o$) Equation (30) gives $\tau \to 2\pi/\lambda_g$, and Equation (43) passes to

$$Z_c \approx \frac{60}{V_f} \ell n\left(\frac{0.358\,\lambda_g}{D}\right). \tag{50}$$

This is to be compared with the Schelkunoff approximate expression evaluated for quarter-wave resonance

$$Z_c = \frac{60}{V_f}\left[\ell n\left(\frac{\lambda_g}{D}\right) - 1\right] \;=\; \frac{60}{V_f} \ell n\left(\frac{0.368\,\lambda_g}{D}\right). \tag{51}$$

So, near resonance, the two formulae differ by only a few percent, which accounts for the success of the prior models.

III. TRANSMISSION LINE MODELING

The formal analysis presented in section II, though fraught with peril, has resulted in a very *practical* model for the RF engineer. The choice of helix geometrical parameters and operating frequency now [by Equation (28)] specify the parameter τ. [Alternatively, by Equation (32), they specify the velocity factor, V_f, and τ is specified by Equation (30).] So we have the axial propagation factor

$$\beta_g = 2\pi/\lambda_g = 2\pi/V_f\,\lambda_0 \tag{52}$$

for the anisotropic helix waveguide. Further, we have its characteristic impedance. [In fact, the entire quest of the

Section II was merely to obtain these two parameters (Z_c and β_g) for the helix.] Consequently, the input and load impedances, for a low-loss line of axial length h, are related in this model by the familiar equation

$$Z_{in} = Z_c \frac{Z_L \cos(\beta_g h) + jZ_c \sin(\beta_g h)}{Z_c \cos(\beta_g h) + jZ_L \sin(\beta_g h)} \quad . \tag{53}$$

The "voltage" is distributed along the line as the *interference pattern of the forward and backward traveling wave pair* that are solutions of the transmission line wave equation and are, for the present, assumed to be monochromatic and coherent. The voltage distribution along a lossy line is given by the familiar expression

$$\begin{aligned} V(x) = &\ V_L \cosh\gamma x \ + \ (I_L Z_c)\sinh\gamma x \\ \approx &\ V_L \left[(1+(Z_c/Z_L)\alpha x)\cos\beta_g x \right. \\ &\left. + j((Z_c/Z_L)+\alpha x)\sin\beta_g x\right] \end{aligned} \tag{54}$$

where the distance x is measured back from the load, V_L is the load voltage, I_L is the load current, α is the attenuation constant, and the other parameters are as defined above. For a low loss, non-radiating, quarter wave line $(\beta_g h = \pi/2)$, open circuited at the load end ($Z_L = \infty$), these give the voltage step-up (or magnification) ratio between the top and bottom of the resonator as[19,20,21,22,23,24]

$$\frac{V_L}{V(-h)} = -j\left(\frac{1}{\alpha h}\right) \tag{55}$$

where $V(-h)$ is the voltage induced in the base of the structure. This is a Tesla coil resonance transformer.[25,26] (Obviously, the load may be a capacitive electrode, which has the dual role of electrically shortening the required structure for system resonance and holding off high voltage discharges until a *desired* potential is attained. It should also be obvious that when one measures the voltage distribution along the structure *under* the electrode, the *voltage* will be a superposition of the actual transmission line standing wave pattern *plus* the inverse r potential of the top electrode.)

By means of conventional distributed-element theory, a thorny boundary value problem has been reduced to a very simple RF transmission line. In fact, the entire design and tuning exercise (see Figure 2) can now be performed conveniently on a Smith chart.[27,28]

IV. TUNED DISTRIBUTED SYSTEMS

We now turn to the tuning of an ensemble of helical resonators. From Equation (54), the voltage distribution along a quarter-wave structure can be approximated by

$$V(x) = V_{Top} \cos\left(\frac{\pi}{2}\frac{x}{h}\right) = V_{Top} \sin\left[\left(1+\frac{x}{h}\right)\frac{\pi}{2}\right] \tag{56}$$

where x is measured back from the load as shown in Fig. 2. The forward and backward traveling waves have superposed

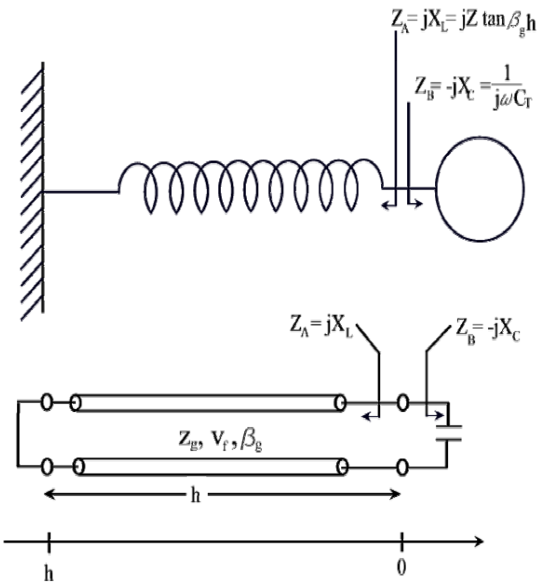

Figure 2. A capacitively tuned distributed resonator.

to give this voltage standing wave distribution along the resonator. There is a voltage null at the base (x = - h), a voltage maximum at the top (x = 0), and a sine wave envelope along the structure. It is of interest to study the behavior of the system when the operating frequency is fixed. As the electrical length of the helical waveguide is shortened from 90° to 15°, the resonating capacitor must be increased in size. (See Figure 3.) Further, the voltage distribution passes from the loop of a sinusoid (at 90°) to the linear portion of the sinusoid (for heights less than 15°. At such short heights, only the first term in the Taylor series expansion for the expression about x = - h is needed, and the voltage rises as it would along the secondary turns of a transformer with uniform current. Under such conditions, one passes to the lumped-element regime and the high VSWR advantage discovered by Tesla is lost.

V. PASSAGE TO LUMPED ELEMENTS

A. Classical Inductors.

Lumped element circuit theory assumes that there are no wave interference phenomena present, that is - the current entering and leaving the circuit element's terminals are identical. This is manifested by two phenomena:

1. The current distribution function is spatially uniform across each element.
2. The spatial phase delay between circuit extremities is zero.

The phase retardation effects of item 2 occur in two ways: first, because c is not really infinite, the time required for an effect to propagate *through space* from one part of a circuit

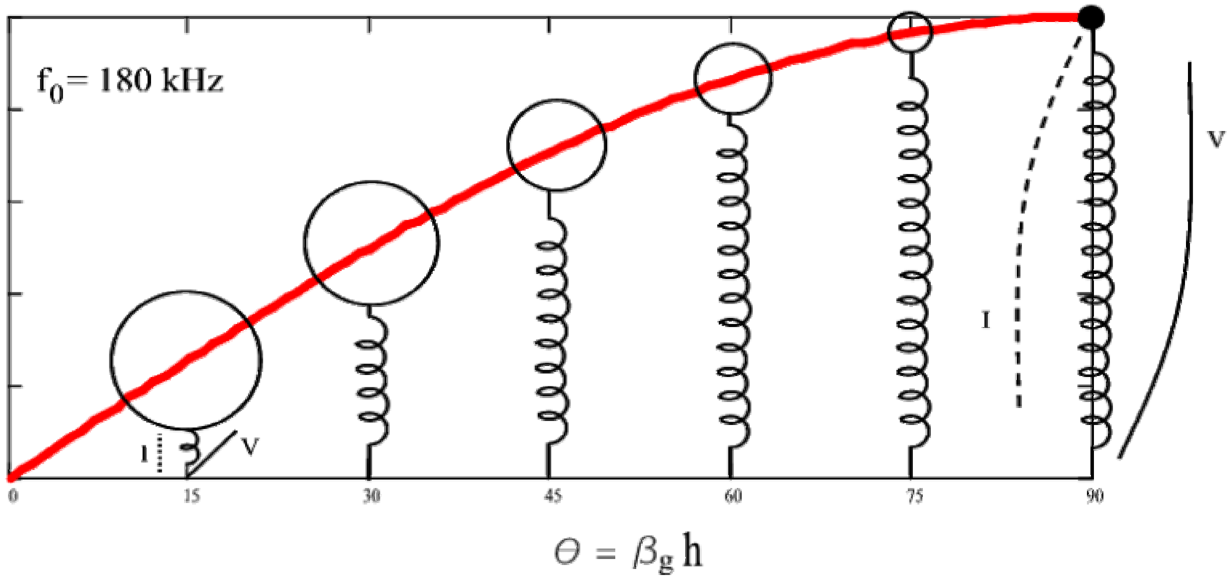

Figure 3. The solid curve is the voltage rise along a quarter wave resonator and shows the passage to lumped elements. As the load capacitance increases (with resonant frequency held constant), the transmission line must be shortened . . . until it behaves as a lumped inductance with a uniform current and linear voltage rise.

to another is appreciable as compared with the period of the changing current; second, the time required for an effect to propagate *along circuit conductors* is appreciable as compared with the period of the current. Ramo and Whinnery observe that item 1 is closely related to item 2, and comment, *"If, at a given instant, the current varies along a path… there must be a temporary piling up, or a decreasing, of charge at various points around the loop."*[29] The current distribution under such circumstances will be spatially nonuniform. (The equation of continuity still is satisfied, of course.) King states, *"All of lumped element electric circuit theory is based upon [these] two inherent assumptions … so much so that it is seldom considered necessary to even mention the existence of such restrictions on the generality of the theory.*"[30] The issue is that these restrictions are often overlooked. Furthermore, the standard handbook formulae for inductances are all based upon the two assumptions. To see this, consider how the standard formula for "inductance" was obtained. The field integral is, indeed, a function of the current distribution, as is the flux. But, under the assumption that $\boldsymbol{I}(\ell') = \boldsymbol{I_o}$ is spatially uniform, one may factor out the current from under the integral sign so that the magnetic flux per unit current is given by the strictly *geometrical* formula of Neumann[31]

$$\boldsymbol{L} \;\equiv\; \frac{\Phi}{\boldsymbol{I_o}} \;=\; \frac{\mu}{4\pi}\oint_{\ell}\oint_{\ell'}\frac{d\vec{\ell}'\bullet d\vec{\ell}}{|\vec{r}-\vec{r}'|}\;. \qquad \textbf{(57)}$$

It is this process (which neglects spatial variations in the current distribution on coils) that has been used in the handbook formulae for self and mutual inductance. Of course, the uniform current assumption has no validity for coils operating anywhere near self-resonance!

Sichak has employed the expression for the helix characteristic impedance and the propagation factor expression, and he makes the following assertion, *"The standard formula for the inductance of a long solenoid can be obtained by treating the solenoid as a short length of short-circuited helical transmission line and using Equation (43) for the characteristic impedance Z_c and the propagation constant from Equation (28).*"[32] That is, since $\tan(x) \approx x$ for small x, the short line's terminal point impedance becomes

$$\boldsymbol{Z_{in}} = \boldsymbol{j\omega L} \;=\; \boldsymbol{jZ_c}\,\tan\!\left(\boldsymbol{\beta_g h}\right) \xrightarrow[\beta_g h \to 0]{} \frac{\boldsymbol{c\beta_g h}}{\boldsymbol{\upsilon_p}}\,60\,\boldsymbol{I}_0(\boldsymbol{\tau a})\,\boldsymbol{K}_0(\boldsymbol{\tau a})\,. \qquad \textbf{(58)}$$

According to Sichak, this passes to the classical lumped element formula for inductance.

B. What About the Issue of Coil Self-Capacitance?

The behavior of distributed networks (such as wires, periodic physical structures, helices, corrugated wave guides, antennas, etc.) may be conveniently *represented* at a pair of terminals by lumped elements. Paris and Hurd have said, *"It is customary in practice to speak of stray or distributed effects when the behavior of a circuit or device cannot be predicted on the basis of ordinary network theory.*"[33] The failure of any lumped element circuit model to describe the real world lies at its core inherent *presupposition*: the speed of light is assumed infinite in the wave equation (all regions of the universe can be communicated with instantaneously[34]). Consequently, lumped element circuit theory does not (and cannot) accurately embody a world of second order partial differential equations in space and time. Lumped elements "have no physical dimensions and no preferred orientation in space; they can be moved around and rotated at will."[35] Not so for real world coils.

What is coil self-capacitance? Physical arguments start by drawing turn-to-turn capacitors and stray capacitances extended out through space to the environment. The matter is concerned with physically resolving the fact that the RF *reactance* of a coil is not that calculated by employing handbook (uniform current) formulae for inductance, nor is it that obtained by measuring L at 1 kHz and multiplying by ω. The concept of coil "self capacitance" is an attempt to circumvent transmission line effects on small coils when the current distribution begins to depart from its DC behavior. The notion has been developed by starting with Maxwell's equations and using only the first two terms in the Taylor series expansion for the distributed current to obtaining an expression for the self-impedance of a generalized closed circuit.[36] Upon extracting Neumann's formula for the self inductance, the remaining negative component of the reactance permits an expression for the coil self-capacitance. These formulae are valid for a *parallel* combination of an inductance and a capacitance when the operating frequency is well below $1/\sqrt{\boldsymbol{LC_L}}$. They permit a coil with a *slightly* nonuniform current distribution to be treated *as though the current were uniform* and the coil was shunted with a lumped element capacitance. There are a great number of formulae for coil self capacitance.[37,38] None are of particular value for quarter-wave helical resonators anywhere near the 90° point. They do have some merit for coils constructed of "many-turns-of-fine-wire" if they are operated well below their self-resonances, but these "induction coils" are *not* really Tesla coils. The practice of using "many turns of fine wire" to construct Tesla coils is self-defeating. Tesla, in fact, abandoned the practice prior to 1893. (By 1897 he was advocating "heavy cable #8".)

Medhurst attempted to characterize a grounded-base coil with a nonuniform current (i.e., one with spatial modes) as a lumped-element inductor (with a known low frequency inductance formula) in parallel with a parasitic, empirically obtained, non-physical, lumped-element capacitor. By constructing many coils, measuring their self-resonant frequencies and low frequency inductances (where I(z) is uniform – and, of course, not the same as at the resonant frequency), he was able to deduce a set of sinusoidal steady-state "self-capacitances". To these he fit a curve that was specified in terms of the coil's length-to-diameter ratio. The empirically obtained "self-capacitance" closely matched the curve

$$\boldsymbol{C_o}(\boldsymbol{h},\boldsymbol{D}) = \left(0.1126\,\frac{\boldsymbol{h}}{\boldsymbol{D}} + 0.08 + \frac{0.27}{\sqrt{\boldsymbol{h}/\boldsymbol{D}}}\right)\boldsymbol{D} \qquad \textbf{(59)}$$

where C_o is in picofarads, h is the axial length (or height) of the coil and D is the coil's diameter (both in cm). Of course, this is merely a statistical determination appropriate for computations in the given h/D regime, and not at all a physical quantity. This is purely an empirical determination for *characterizing* the impedance of a structure *at a pair of terminals*. The intent was to represent the true coil (which has a nonuniform current distribution) by a lumped element coil (with an *assumed* uniform current distribution) in parallel with a non-physical effective capacitance. The formula will *not* (and, being lumped, can not) give the voltage magnification by VSWR due to physically true current standing waves on the structure but it does permit a terminal point representation for the resonator's *feed point* reactance. The same (or an even better) formula could be obtained from the field solution by performing "numerical experiments" with Equations (43) and (53). Figure 4 compares the exact field solution, Medhurst's approximation, and the lumped element representation for the reactance of a coil as a function of its axial length. If impedance is the only item of interest, the empirical Medhurst approximation is acceptable out to about 60º.

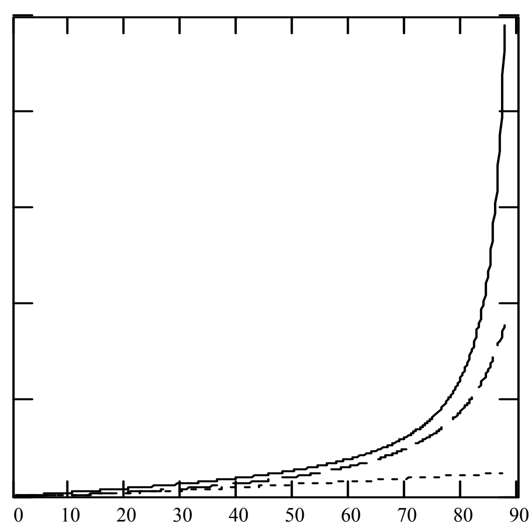

Figure 4. Graph of terminal point inductive reactance as a function of coil electrical length in degrees: true (solid), Medhurst's empirical parasitic element characterization (dashed), and conventional lumped element formula (dotted).

VI. DISCUSSION

A. Performance Parameters

It has been asserted by some that the performance of a Tesla coil is to be measured by the ratio of discharge length to the Tesla coil height. From Figure 3, this is a delusion. By that measure, (since electrical breakdown is inversely proportional to electrode curvature – the bigger the ball, the smaller the curvature and the higher the breakdown potential) one would make the coil very short so that the top electrode could be immense – and the breakdown potential very large. The delusion is that the short coil is then made to operate in the lumped element regime (where the current distribution is uniform) and the voltage rise is limited to no more than that possible from the expression

$$V_2 = V_1 \sqrt{C_1/C_2} \tag{60}$$

which, not only, presupposes lumped elements, but also total energy transfer to the secondary (i.e., correct switching durations), and lossless systems. With this misguided philosophy, a far greater amount of energy is required to bring the electrode up to breakdown and match the performance of a relatively simple distributed resonator operating in the standing wave regime. Tesla has commented upon this misuse of helical coil resonators, and states, *"A large capacity and a small self inductance is the poorest kind of circuit which can be constructed."*[39] It should now be clear why Tesla was right and this is true. The breakdown is set by the electrode size. Because, voltage rise is proportional to $1/\alpha\ell$ (a genuine performance parameter for resonators), the same potential can be attained with far less energy by using standing waves on a system that is, say, electrically 75º tall than one that is merely 10º tall and operating in the lumped element regime. While we know that nature does not deal in infinities, we can now appreciate why Tesla would say, *"With such coils, I have found that there was practically no limit to the tension available,"*[40] and on another occasion, *"I have produced electrical discharges the actual path of which, from end to end, was probably more than 100 feet long; but it would not be difficult to reach lengths one hundred times as great."*[41] While Equation (60) is bounded, Equation (55) gives the fundamental limit for which to strive for voltage step-up.

Virtually all high performance Tesla coils are velocity inhibited, distributed-element, slow wave transmission line helical resonators. By the way, Tesla said that he discovered this striking nature of RF coils experimentally in 1894, *"That was the first single step toward ... my magnifying transmitter."*[42] Interestingly, the above analysis also holds when the helical coil is pulled out into a linear conductor. Examples of this are given by quarter-wave coaxial transmission line resonators (unbalanced "monopole-mode" Tesla coils) and parallel-wire transmission line resonators (balanced "dipole-mode" Tesla coils).

B. Fundamental Limits

The extension of the present analysis to quasimonochromatic, partially coherent wave distributions should be self-evident. The key performance parameter for high voltage Tesla coils is the VSWR on the resonant structure itself – *the higher the better!* This depends upon the ability of the forward and backward waves to constructively interfere, and it is intimately related to the fringe "Visibility Function" of partially coherent beams in optics.[43] In fact, a true Tesla coil is a slow-wave transmission line analog of a Fabry-Perot resonator/interferometer with a conducting surface at one plane and a high impedance interface at the other. With LC-discharge excited Tesla coils, it is possible to have such a poorly designed resonator (as in the "many turns of fine wire" philosophy) that the forward and backward waves are so uncorrelated that the voltage distribution is virtually uniform – no minima and no maxima over the structure!

C. Other Models for the Helix.

In addition to the helix characteristic impedance employed above, there are a variety of wave impedance parameters that have been developed that are of some interest for applications to traveling wave tubes and even applied to model coils.[44] Further, the helix analysis above has been based upon Ollendorf's helix model, which represents the helically wound coil as an anisotropic sheath conductance. There are other helix models. The thin-wire model of Kogan can be derived from either the electric Hertz vector or from the vector potential.[45,46] Bondar also studied the thin wire model.[47] The tape helix model of Sensiper[48] brings out many of the periodic properties of wave propagation on helices. It is unfortunate that we have not space in this present note to present experimental measurements. Such a note was prepared for Tesla coil experimenters, and printed several years ago in the literature for hobbyists.[49]

D. Polychromatic Excitation

We note that the discussion above (including the work of Medhurst), was framed within the sinusoidal steady state. Many experimentalists excite their coils with transient pulses, and such waveforms are *finite energy* signals, while periodic waveforms are *finite power* signals. Conceptually (and practically) the passage to a pulse-type (finite energy) wide-band response may be obtained in terms of the *synthetic pulse* idea: given the Fourier spectrum of the transient excitation waveform, determine the system response to each separate spectral component and then combine to obtain the result.[50] This is particularly easy to do for high Q resonators since the coherence time of waves on the structure is so long. The canonical variables are, essentially, adiabatic invariants.

VII. CONCLUSIONS

The present note has modeled RF coils as slow-wave anisotropic waveguides. A solution of the boundary value problem has given not only the fields, but also the eigenvalue equation for the propagation parameter (τ), the velocity factor (V_f), the wave effective characteristic impedance (Z_c), and the limiting voltage magnification $\left(1/\alpha h\right)$ caused by wave interference or cavity modes. These parameters permit a comprehensive engineering description of helix design as a simple surface-wave transmission line, and they are appropriate for designing and tuning Tesla coil helical resonators on conventional Smith charts. Further, this development analytically clarifies the smooth conceptual transition from field theory - to distributed elements - to lumped elements.

ACKNOWLEDGEMENT

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